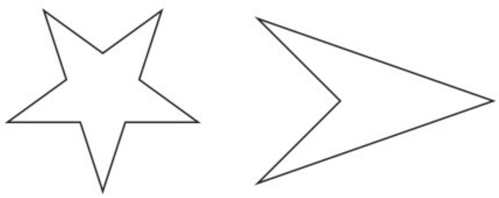
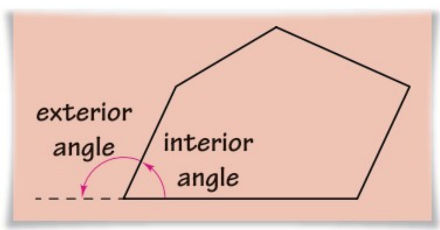
2.4 The Ins and Outs of Polygons

Familiar figures like triangles, parallelograms, and trapezoids are called **convex polygons.** Figures like the star and the arrowhead pictured here are called **concave polygons.**



For convex polygons it is clear which points are on the inside and which are on the outside. It is also clear how to measure the **interior angles.**

By extending a side of a convex polygon, you can make an **exterior angle** that lies outside the polygon.



The figures below show two ways to form exterior angles. You can extend the sides as you move in either direction around the polygon.

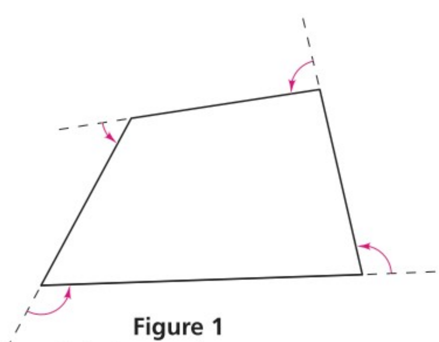


Figure 1
Exterior angles as you move counterclockwise.

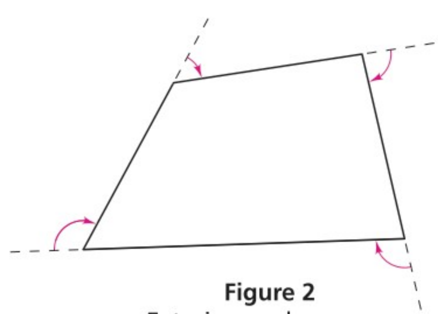


Figure 2
Exterior angles as you move clockwise.

Measuring exterior angles provides useful information about the interior angles of a polygon.

Problem 2.4

Members of the Columbia Triathlon Club train by bicycling around the polygonal path shown.

They start at vertex *A* and go on to vertices *B*, *C*, *D*, and *E*. Then they return to *A* and start another lap. At each vertex the cyclists have to make a left turn through an *exterior angle* of the polygon.

A

1. What is the sum of the left-turn exterior angles that the cyclists make on one full lap around this path?
   - Explain how you can arrive at an answer without measuring.
   - Then, measure the exterior angles to check your thinking.
2. Draw several other polygons. Include a triangle, a quadrilateral, and a hexagon. Find the sums of the turn angles if you cycle around each figure and return to your start point and direction.
3. Will the turning pattern you observed in cycling around several polygons occur in any other polygons? Why or why not?

Problem 2.4 continued

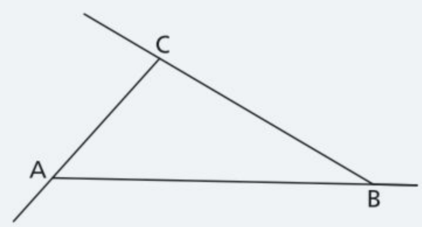
Each exterior angle and its adjacent interior angle are *supplementary angles*.

B 1. Consider the polygonal training track shown on the previous page. How many pairs of supplementary angles are there?

2. Amy says there are 5 straight angles in the diagram. They total $T = 5 \cdot 180°$. She thinks there is way to figure out the part of T that is the sum of the interior angles. She also wants the part of T that is the sum of the exterior angles. How can she find each part of T?

3. Becky says $T = n \times 180°$ should work for the total of exterior and interior angles for any polygon. So $n \times 180° - 360°$ should give her the sum of the interior angles of any polygon. But this does not look like the formula she found in Problem 2.2. Use the formula you developed in Problem 2.2. Explain to Becky why her formula is equivalent.

C Nic thought about exterior angles and 'walking around' a polygon. He came up with a new way to prove that the sum of the interior angles of any triangle is 180°. Answer Nic's questions that follow to complete his proof.



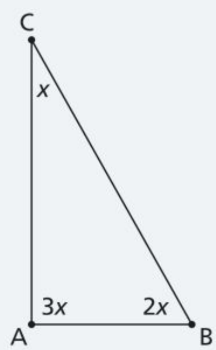
1. What is the sum of all of the interior and exterior angles in any triangle?

2. What is the sum of the exterior angles?

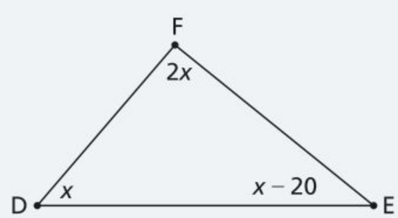
3. How much is left for the sum of the interior angles?

D For each of the following triangles write and solve an equation to find the value of x. Use the results to find the size of each angle. Find the supplement of each interior angle.

1.



2.



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