

Investigation

2

Designing Polygons: The Angle Connection

This Investigation develops properties of polygons. These properties make polygons useful in many natural objects like the combs made by bees to store their honey.

2.1 Angle Sums of Regular Polygons

You have seen that polygons with the same number of sides can have different shapes. However, there is an important relationship between the number of sides and the angle sum of any polygon. You will develop a formula that relates the number of sides to angle measures.

A **regular polygon** is a polygon in which all of the sides are the same length and all of the angles have the same measure. In an **irregular polygon**, not all of the sides are the same length or not all of the angles have the same measure.

Polygons are named based on the number of sides and angles they have. For example, a polygon with six sides and six angles is called a *hexagon*.

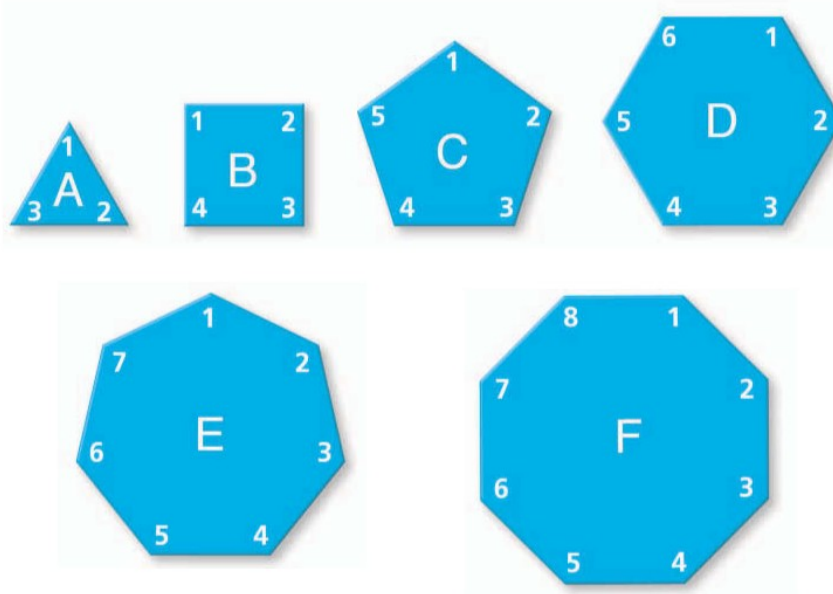
Common Core State Standards

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

7.EE.B.4 Use variables to represent quantities in a real world or mathematical problem, and construct simple equations . . . to solve problems by reasoning about the quantities.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Below are six examples of polygons from the Shapes Set. Study these examples to find a relationship between the number of sides and angles.



? Is there a relationship between the size of the angles and the number of sides for regular polygons?

Problem 2.1



You can discover relationships between the number of sides and the angle measures of polygons. Measure some examples, organize the data, and then look for patterns.

- A** Use an angle ruler to measure the angles in the equilateral triangle, the square, the regular pentagon, and the regular hexagon from the Shapes Set.

1. Enter the results in a table like that begun here.

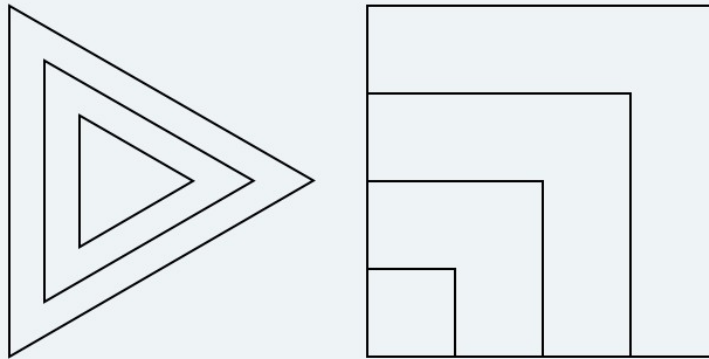
Polygon	Number of Sides	Measure of an Angle	Angle Sum
Triangle	■	■	■
Square	■	■	■
Pentagon	■	■	■
Hexagon	■	■	■
Heptagon	■	■	■
Octagon	■	■	■
Nonagon	■	■	■
Decagon	■	■	■

2. Find a pattern that suggests a way to fill in the table for regular polygons with seven, eight, nine, and ten sides. Then measure the angles of the Shapes Set heptagon and octagon. See if your pattern holds in those cases.
3. Describe a pattern relating angle sums to number of sides in regular polygons.
4. Describe a pattern relating measures of individual angles and number of sides in regular polygons.

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Problem 2.1 *continued*

- B** The diagram below shows two sets of regular polygons of different sizes. Does the pattern relating number of sides, measures of angles, and angle sums apply to all of these shapes? Explain your reasoning.



- C** Explain how you could find the angle sum of a regular polygon with n sides. Then, write your conjecture as a formula $S = \underline{\hspace{2cm}}$. The right side of the equation should give an expression for calculating the sum from the value of n .
- D** Explain how you could find the measure of each angle in a regular polygon with n sides. Then, write your conjecture as a formula $A = \underline{\hspace{2cm}}$. The right side of the equation should give an expression for calculating the measure of each angle from the value of n .

A C E Homework starts on page 52.