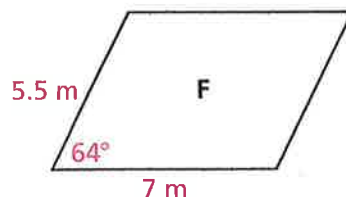
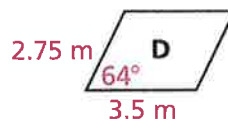
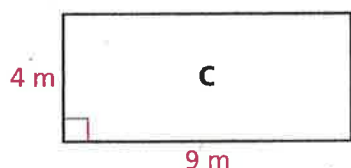
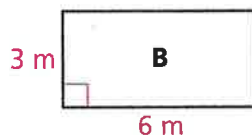
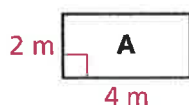




Applications

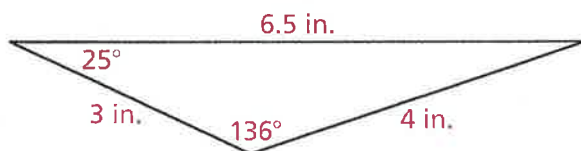
1. For parts (a)–(c), use the parallelograms below.



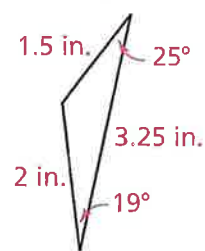
- a. List all the pairs of similar parallelograms. Explain your reasoning.
 - b. For each pair of similar parallelograms, find the ratio of two adjacent side lengths in one parallelogram. Find the ratio of the corresponding side lengths in the other parallelogram. How do these ratios compare?
 - c. For each pair of similar parallelograms, find the scale factor from one shape to the other. Explain how the information given by the scale factors is different from the information given by the ratios of adjacent side lengths.
2.
 - a. On grid paper, draw two similar rectangles where the scale factor from one rectangle to the other is 2.5. Label the length and width of each rectangle.
 - b. For each rectangle, find the ratio of the length to the width.
 - c. Draw a third rectangle that is similar to one of the rectangles in part (a). Find the scale factor from the new rectangle to the one from part (a).
 - d. Find the ratio of the length to the width for the new rectangle.
 - e. What can you say about the length-to-width ratios of the three rectangles? Is this true for another rectangle that is similar to one of the three rectangles? Explain.

3. For parts (a)–(d), use the triangles below. The drawings are not to scale.

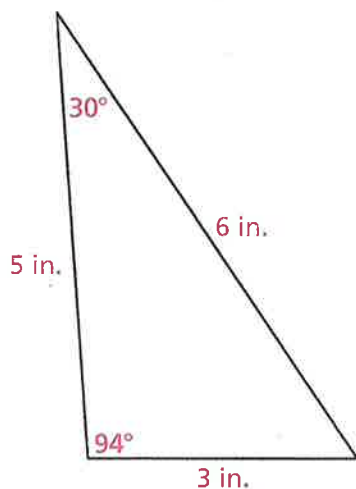
Triangle A



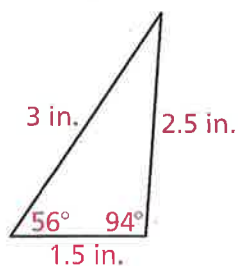
Triangle B



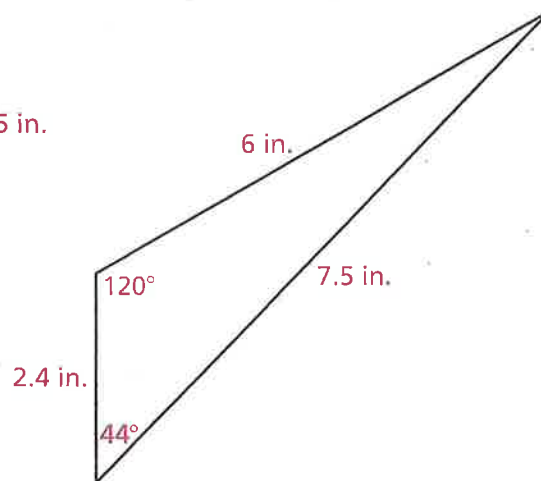
Triangle C



Triangle D



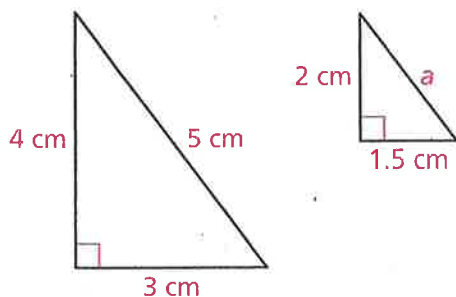
Triangle E



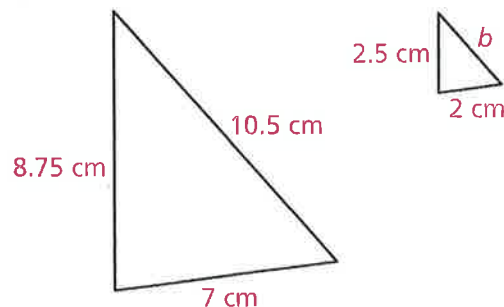
- List all the pairs of similar triangles. Explain why they are similar.
- For each pair of similar triangles, find the ratio of two side lengths in one triangle. Find the ratio of the corresponding side lengths in the other. How do these ratios compare?
- For each pair of similar triangles, find the scale factor from one shape to the other. Explain how the information given by the scale factors is different than the information given by the ratios of side lengths.
- How are corresponding angles related in similar triangles? Is it the same relationship as for corresponding side lengths? Explain.

For Exercises 4–7, each pair of figures is similar. Find the missing measurement. Explain your reasoning. (Note: The figures are not drawn to scale.)

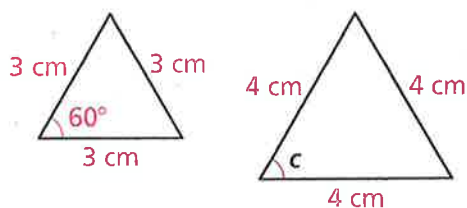
4.



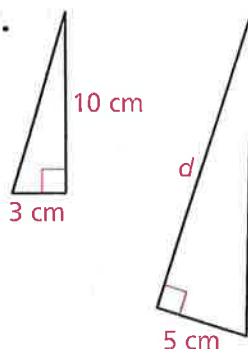
5.



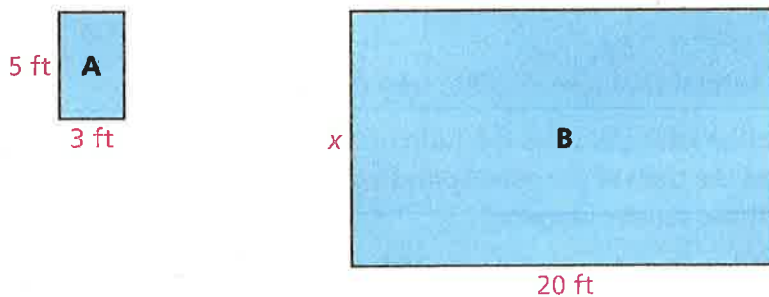
6.



7.



For Exercises 8–10, Rectangles A and B are similar.



8. **Multiple Choice** What is the value of x ?

A. 4

B. 12

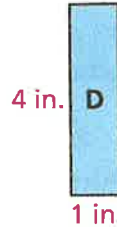
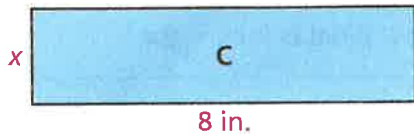
C. 15

D. $33\frac{1}{3}$

9. What is the scale factor from Rectangle B to Rectangle A?

10. Find the area of each rectangle. How are the areas related?

11. Rectangles C and D are similar.



- What is the value of x ?
 - What is the scale factor from Rectangle C to Rectangle D?
 - Find the area of each rectangle. How are the areas related?
12. Suppose you want to buy new carpeting for your bedroom. The bedroom floor is a 9-foot-by-12-foot rectangle. Carpeting is sold by the square yard.
- How much carpeting do you need to buy?
 - Carpeting costs \$22 per square yard. How much will the carpet cost?
13. Suppose you want to buy the carpet described in Exercise 12 for a library. The library floor is similar to the floor of the 9-foot-by-12-foot bedroom. The scale factor from the bedroom to the library is 2.5.
- What are the dimensions of the library? Explain.
 - How much carpeting do you need for the library?
 - How much will the carpet for the library cost?
14. The Washington Monument is the tallest structure in Washington, D.C. At a certain time, the monument casts a shadow that is about 500 feet long. At the same time, a 40-foot flagpole nearby casts a shadow that is about 36 feet long. About how tall is the monument? Sketch a diagram.



15. Darius uses the shadow method to estimate the height of a flagpole. He finds that a 5-foot stick casts a 4-foot shadow. At the same time, he finds that the flagpole casts a 20-foot shadow. What is the height of the flagpole? Sketch a diagram.
16. a. Greg and Zola are trying to find the height of their school building. Zola takes a picture of Greg standing next to the building. How might this picture help them determine the height of the building?
- b. Greg is 5 feet tall. The picture Zola took shows Greg as $\frac{1}{4}$ inch tall. If the building is 25 feet tall in real life, how tall should the building be in the picture? Explain.
- c. In part (a), you thought of ways to use a picture to find the height of an object. Think of an object in your school that is difficult to measure directly, such as a high wall, bookshelf, or trophy case. Describe how you might find the height of the object.
17. Movie screens often have an *aspect ratio* of 16 by 9. This means that for every 16 feet of width along the base of the screen there are 9 feet of height. The width of the screen at a local drive-in theater is about 115 feet wide. The screen has a 16 : 9 aspect ratio. About how tall is the screen?
18. Triangle A has sides that measure 4 inches, 5 inches, and 6 inches. Triangle B has sides that measure 8 feet, 10 feet, and 12 feet. Taylor and Landon are discussing whether the two triangles are similar. Do you agree with Taylor or with Landon? Explain.

Taylor's Explanation

The triangles are similar. If you double each of the side lengths of Triangle A, you get the side lengths for Triangle B.

OR

Landon's Explanation

The triangles are not similar. Taylor's method works when the two measures have the same units. However, the sides of Triangle A are measured in inches, and the sides of Triangle B are measured in feet. So, they cannot be similar.

Connections



For Exercises 19–24, tell whether each pair of ratios is equivalent.

19. 3 to 2 and 5 to 4

20. 8 to 4 and 12 to 8

21. 7 to 5 and 21 to 15

22. 1.5 to 0.5 and 6 to 2

23. 1 to 2 and 3.5 to 6

24. 2 to 3 and 4 to 6

25. Use a pair of equivalent ratios from Exercises 19–24. Write a similarity problem using the ratios. Explain how to solve the problem.

For each ratio in Exercises 26–29, write two other equivalent ratios.

26. 5 to 3

27. 4 to 1

28. 3 to 7

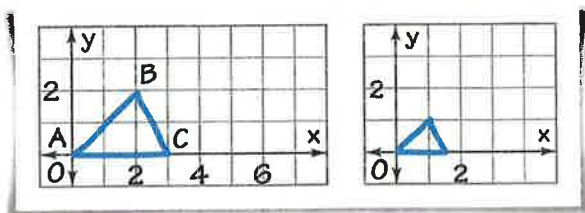
29. 1.5 to 1

30. Here is a picture of Duke. The scale factor from Duke to the picture is 12.5%. Use an inch ruler to make any measurements.



- How long is Duke from his nose to the tip of his tail? Explain how you used the picture to find your answer.
- To build a doghouse for Duke, you need to know his height. How tall is Duke? Explain.
- A copy center has a machine that prints on poster-size paper. You can resize an image from 50% to 200%. How can you use the machine to make a life-size picture of Duke?

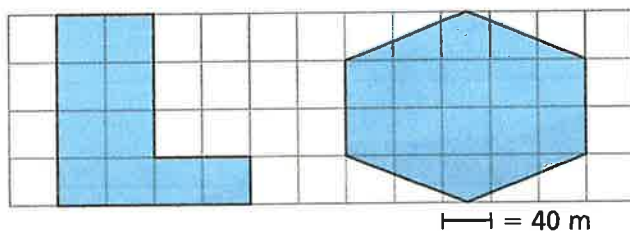
31. Paloma draws triangle ABC on a grid. She applies a rule to make the triangle on the right.



- What rule did Paloma apply to make the new triangle?
- Is the new triangle similar to triangle ABC ? Explain your reasoning. If the triangles are similar, give the scale factor from triangle ABC to the new triangle.

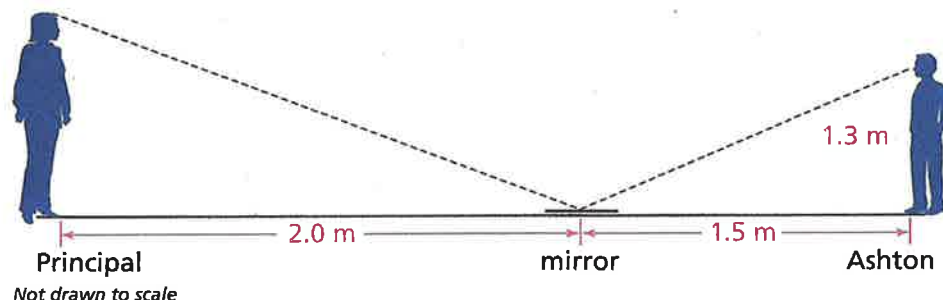
For Exercises 32 and 33, use the paragraph below.

The Rosavilla School District wants to build a new middle school building. They ask architects to make scale drawings of possible layouts for the building. Two possibilities are shown below.



- What is the area of each scale drawing in square units?
 - What would the area of the ground floor of each building be?
- Multiple Choice** The school board likes the L-shaped layout but wants a building with more space. They increase the L-shaped layout by a scale factor of 2. For the new layout, choose the correct statement.
 - The area is two times the original.
 - The area is four times the original.
 - The area is eight times the original.
 - None of the statements above are correct.

34. The school principal visits Ashton's class one day. Ashton uses the mirror method to estimate the principal's height. This diagram shows the measurements he records.



- What estimate should Ashton give for the principal's height?
 - Is your answer to part (a) a reasonable height for an adult?
35. Use the table for parts (a)–(c).

Student Heights and Arm Spans

Height (in.)	60	65	63	50	58	66	60	63	67	65
Arm Span (in.)	55	60	60	48	60	65	60	67	62	70

- Find the ratio of arm span to height for each student. Write the ratio as a fraction. Then write the ratio as an equivalent decimal. What patterns do you notice?
 - Find the mean of the ratios.
 - Use your answer from part (b). Predict the arm span of a person who is 62 inches tall. Explain your reasoning.
36. For each angle measure, find the measure of its complement and the measure of its supplement.

Sample: 30° complement: 60° ; supplement: 150°

a. 20°

b. 70°

c. 45°



37. The rectangles at the right are similar.

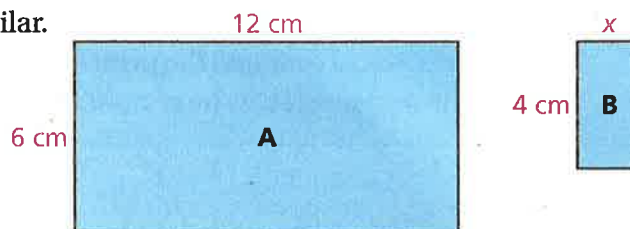
a. What is the scale factor from Rectangle A to Rectangle B?

b. Complete the following sentence in two different ways. Use the side lengths of Rectangles A and B.

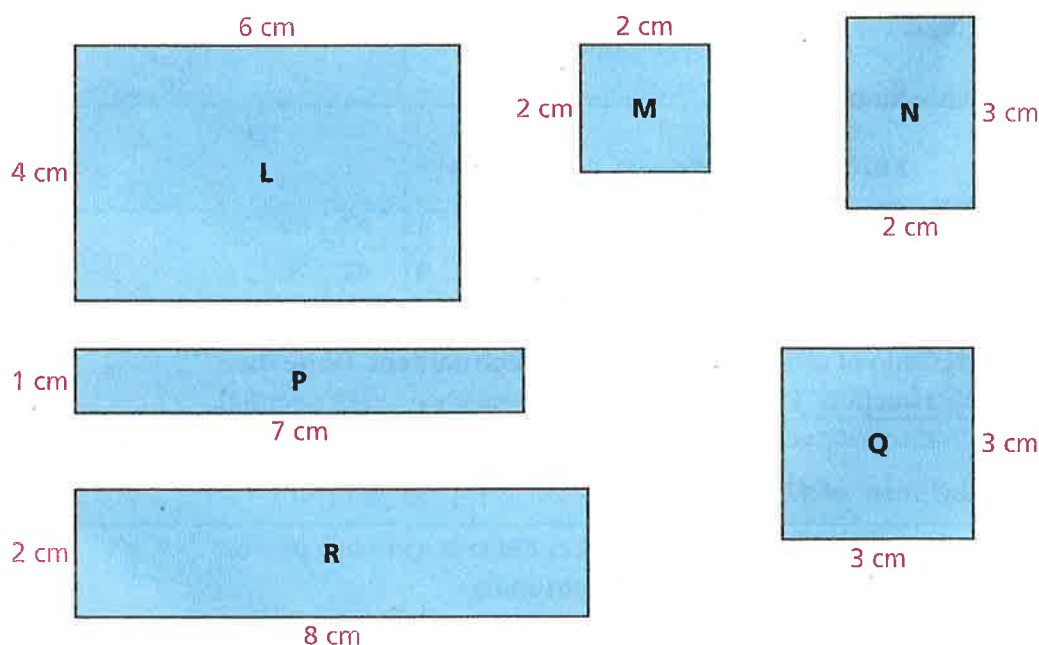
The ratio of \square to \square is equivalent to the ratio of \square to \square .

c. What is the value of x ? Explain your reasoning.

d. What is the ratio of the area of Rectangle A to the area of Rectangle B?



For Exercises 38 and 39, use the rectangles below.



38. **Multiple Choice** Which pair of rectangles listed below is similar?

A. L and M

B. L and Q

C. L and N

D. P and R

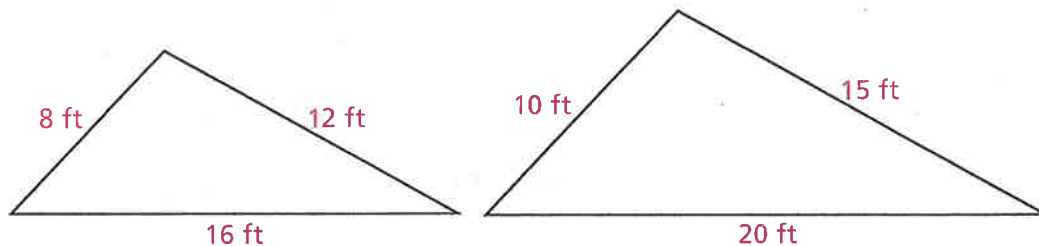
39. a. Find at least one more pair of similar rectangles.

b. For each pair of similar rectangles, find the scale factor from the larger rectangle to the smaller rectangle. Find the scale factor from the smaller rectangle to the larger rectangle.

c. For each similar pair of rectangles, find the ratio of the area of the larger rectangle to the area of the smaller rectangle.

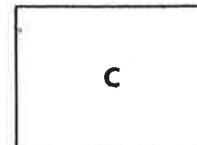
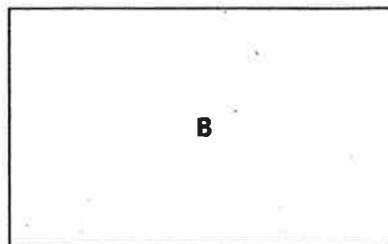
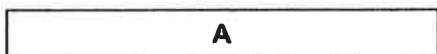
Extensions

40. For parts (a)–(e), use the similar triangles below.



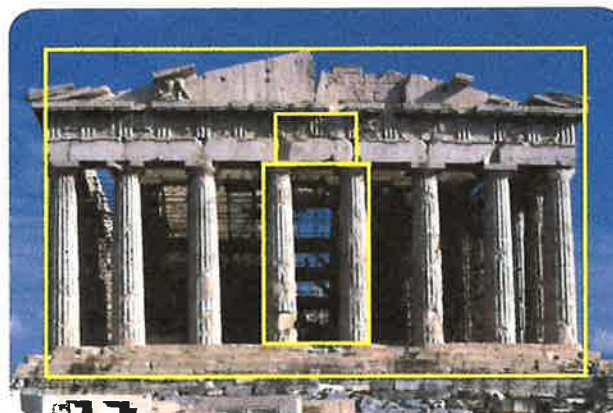
- What is the scale factor from the smaller triangle to the larger triangle? Write your answer as a fraction and a decimal.
 - Choose any side of the larger triangle. Find the ratio of this side length to the corresponding side length in the smaller triangle. Write your answer as a fraction and as a decimal. How does the ratio compare to the scale factor from part (a)?
 - What is the scale factor from the larger triangle to the smaller triangle? Write your answer as a fraction and a decimal.
 - Choose any side of the smaller triangle. Find the ratio of this side length to the corresponding side length in the larger triangle. Write your answer as a fraction and as a decimal. How does the ratio compare to the scale factor from part (c)?
 - What patterns do you notice in parts (a)–(d)? Are these patterns the same for any pair of similar figures? Explain.
41. For parts (a) and (b), use a straightedge and an angle ruler or protractor.
- Draw two different triangles that each have angle measures of 30° , 60° , and 90° . Do the triangles appear to be similar?
 - Draw two different triangles that each have angle measures of 40° , 80° , and 60° . Do the triangles appear to be similar?
 - Based on your findings for parts (a) and (b), make a conjecture about triangles with congruent angle measures.

42. One of these rectangles is “most pleasing to the eye.”



The question of what shapes are most attractive has interested builders, artists, and craftspeople for thousands of years.

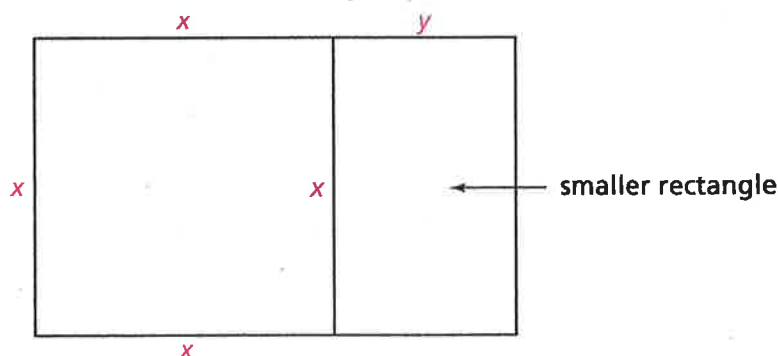
The ancient Greeks were particularly attracted to rectangular shapes similar to Rectangle B above. They referred to such shapes as “golden rectangles.” They used golden rectangles frequently in buildings and monuments. The ratio of the length to the width in a golden rectangle is called the “golden ratio.”



This photograph of the Parthenon (a temple in Athens, Greece) shows several golden rectangles.

- Measure the length and width of Rectangles A, B, and C above in centimeters. For each rectangle, estimate the ratio of the length to the width as accurately as possible. The ratio for Rectangle B is an approximation of the golden ratio.
- You can divide a golden rectangle into a square and a smaller rectangle similar to the original rectangle.

Golden Rectangle

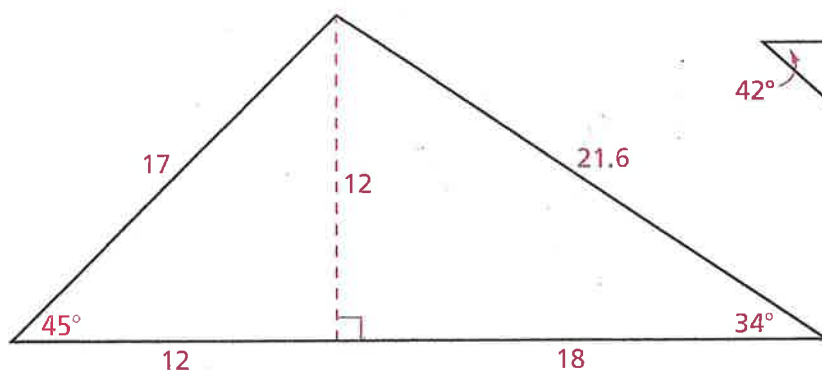


The smaller rectangle is similar to the larger rectangle.

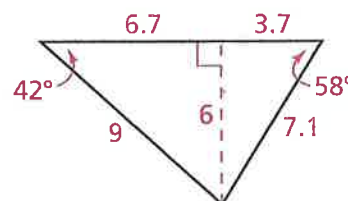
Copy Rectangle B above. Divide this golden rectangle into a square and a rectangle. Is the smaller rectangle a golden rectangle? Explain.

43. For parts (a) and (b), use the triangles below.

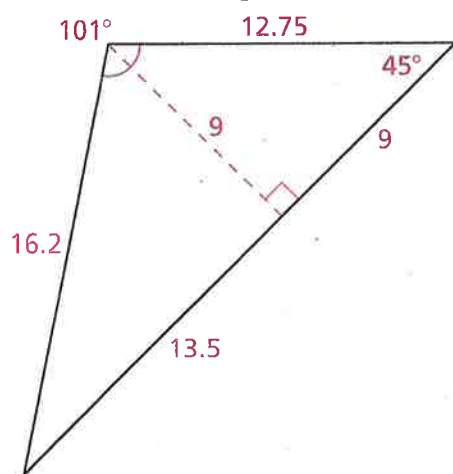
Triangle A



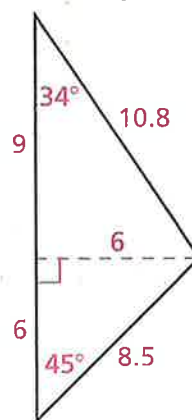
Triangle B



Triangle C



Triangle D



- Identify the triangles that are similar to each other. Explain your reasoning.
- For each triangle, find the ratio of the base to the height. How do these ratios compare for the similar triangles? How do these ratios compare for the non-similar triangles?

For Exercises 44–48, suppose a photographer for the school newspaper took this picture. The editors want to resize the photo to fit in a specific space on a page.



44. Can the original photo be changed to a similar rectangle with the given measurements (in inches)?
- a. 8 by 12 b. 9 by 11 c. 6 by 9 d. 3 by 4.5
45. Suppose that the school copier only has three paper sizes (in inches): $8\frac{1}{2}$ by 11, 11 by 14, and 11 by 17. You can enlarge or reduce documents by specifying a percent from 50 % to 200 %. Can you make copies of the photo that fit exactly on any of the three paper sizes? Explain your reasoning.
46. A copy machine accepts scale factors from 50 % to 200 %. How can you use the copy machine to produce a copy that is 25 % of the original photo's size? How does the area of the copy relate to the area of the original photo?
47. How can you use the copy machine to reduce the photo to a copy that is 12.5 % of the original photo's size? 36 % of the original photo's size? How does the area of the reduced figure compare to the area of the original in each case?
48. What is the greatest enlargement of the photo that will fit on paper that is 11 inches by 17 inches?

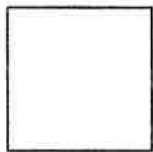
49. The following sequence of numbers is called the *Fibonacci sequence*. It is named after an Italian mathematician from the 14th century who contributed to the early development of algebra.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377 . . .

- Look for patterns in this sequence. How are the numbers found? Use your ideas to find the next four terms.
 - Find the ratio of each term to the term before it. For example, 1 to 1, 2 to 1, 3 to 2, and so on. Write each of the ratios as a fraction and as an equivalent decimal. Compare the results to the golden ratios you found in Exercise 44. Describe similarities and differences.
50. Francisco, Katya, and Peter notice that all squares are similar. They wonder if other shapes that have four sides are *all-similar*. Who is correct?

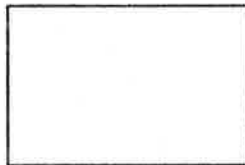
Francisco's Work

Squares are the only type of *all-similar* polygon with four sides. This is because all the sides have equal length, and all the angles are right angles.



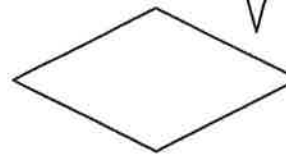
Katya's Work

All rectangles are *all-similar*. Just like squares, all the angles in rectangles are congruent.



Peter's Work

I know that rhombi are four-sided shapes with sides that are all the same length. Rhombi must be *all-similar* because, for two rhombi, there is a consistent scale factor for all corresponding side lengths.



51. Ernie and Vernon are having a discussion about *all-similar* shapes. Ernie says that regular polygons and circles are the only types of *all-similar* shapes. Vernon claims isosceles right triangles are *all-similar*, but they are not regular polygons. Who is correct? Explain.