1.4 Keeping Things in Proportion

Scaling to Solve Proportions

In Problem 1.3 you used ratios and scaling to solve problems. When you write two equivalent ratios in fraction form and set them equal to each other, you form a **proportion.**

Otis's strategy for solving a problem involving a ratio of orange concentrate to juice was to write this proportion:

$$\frac{1}{4} = \frac{x}{128}$$

- Would it have made sense for Otis to write $\frac{1}{x} = \frac{4}{128}$?
- What are some other ways Otis might have written a proportion?
- Otis solved the proportion $\frac{1}{4} = \frac{x}{128}$ by scaling up. He wrote $\frac{1 \cdot 32}{4 \cdot 32} = \frac{x}{128}$.
- How did he know to scale up by $\frac{32}{32}$?

In *Stretching and Shrinking,* you worked with ratios to find missing lengths in similar figures. There are many other situations in which setting up a proportion can help you solve a problem. For example, suppose that among American doctors men outnumber women by a ratio of 12 to 5.

- If about 600,000 American doctors are men, how can you figure out how many are women?

There are four ways to write this as a proportion.

Write the known ratio of men to women doctors. Complete the proportion with the ratio of actual numbers of doctors.

$$\frac{12 \text{ men}}{5 \text{ women}} = \frac{600{,}000 \text{ men}}{x \text{ women}}$$

Write a ratio of men to men data. Complete the proportion with women to women data.

$$\frac{12 \text{ men}}{600{,}000 \text{ men}} = \frac{5 \text{ women}}{x \text{ women}}$$

Write the known ratio of women to men doctors. Complete the proportion with the ratio of actual numbers of doctors.

$$\frac{5 \text{ women}}{12 \text{ men}} = \frac{x \text{ women}}{600{,}000 \text{ men}}$$

Write a different ratio of men to men data. Complete the proportion with women to women data.

$$\frac{600{,}000 \text{ men}}{12 \text{ men}} = \frac{x \text{ women}}{5 \text{ women}}$$

Using what you know about equivalent ratios, you can find the number of women doctors from any one of these proportions. Finding the missing value in a proportion is called *solving the proportion*.

- Does one of the proportions seem easier to solve than the others?
- How many women doctors are there?

Problem 1.4

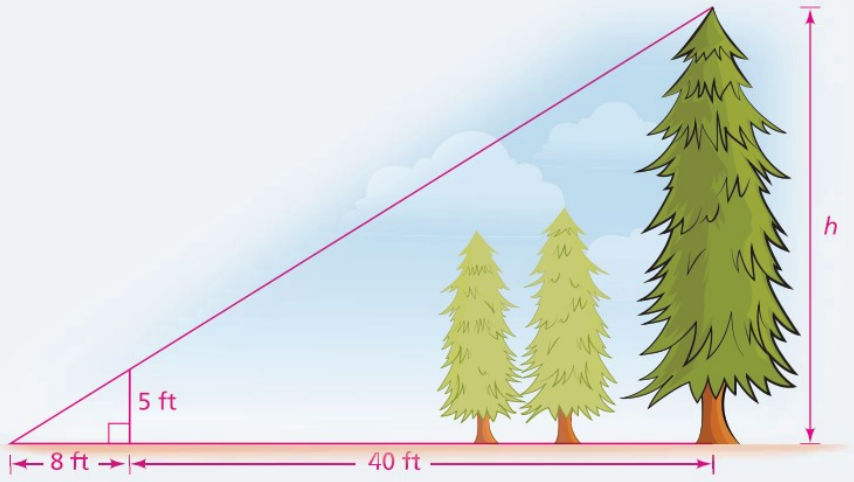
For each question, set up a proportion that shows the relationship between known and unknown quantities. Then use equivalent fractions, ratios, and scaling to solve each proportion.

A Imani gives vitamins to her dogs. The recommended dosage is 1 teaspoon per day for adult dogs weighing 10 pounds. She needs to give vitamins to Bruiser, who weighs 80 pounds and to Dust Ball, who weighs 7 pounds. What is the correct dosage for each dog?

B

1. Jogging 5 miles burns about 500 Calories. How many miles does Tanisha need to jog to burn off the 1,200-Calorie lunch she ate?
2. Tanisha jogs about 8 miles in 2 hours. How long will it take her to jog 12 miles?

C The triangles in this picture are similar. Find the height of the tree.



continued on the next page >

Problem 1.4 continued

D Solve these proportions for the variable x. Use the reasoning you applied in Questions A through C.

1. $\frac{8}{5} = \frac{32}{x}$
2. $\frac{7}{12} = \frac{x}{9}$
3. $25 : x = 5 : 7$
4. $\frac{x}{3} = \frac{8}{9}$
5. $\frac{x}{5} = \frac{120}{3}$
6. $x : 6 = 10 : 150$

E 1. Nic was working on the proportion below.

$$\frac{3}{10} = \frac{x}{6}$$

He could not see a way to scale 10 to make 6. Instead, he scaled both sides of the proportion. His work is shown below. How could Nic complete his solution?

$$\frac{3}{10} = \frac{x}{6}$$

$$\frac{3 \cdot 6}{10 \cdot 6} = \frac{10 \cdot x}{10 \cdot 6}$$

$$\frac{18}{60} = \frac{10x}{60}$$

2. Kevin thinks Nic's idea is great, but he used 30 as a common denominator. Show what Kevin's version of the proportion would look like. Does Kevin's scaled-up proportion give the same answer as Nic's? Explain your reasoning.
3. Does Kevin's work help you solve $\frac{7}{12} = \frac{x}{9}$? Explain.

ACE Homework starts on page 19.