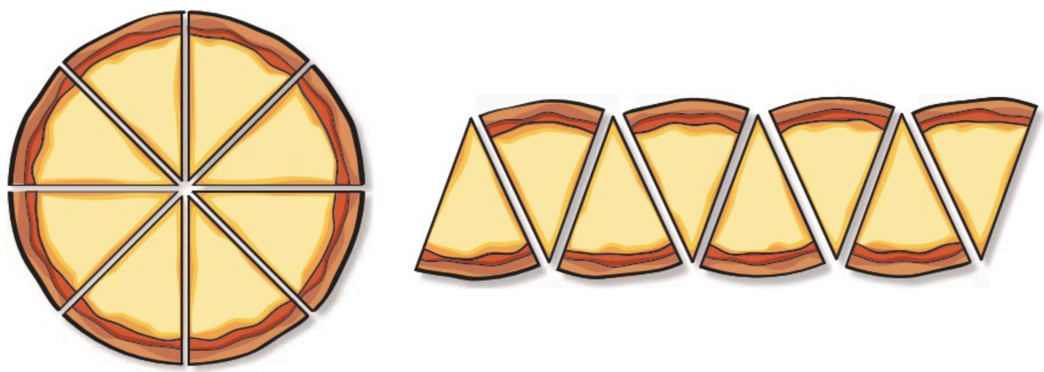
3.4 Connecting Circumference and Area

The special number $\pi \approx 3.14159\ldots$ plays a central role in calculating both circumferences and areas of circles. The rules for finding circumference and area of any circle are expressed in algebraic symbols by the formulas $C = \pi d$, or $C = 2\pi r$, and $A = \pi r^2$. Circumference and area tell quite different things about a circle. However, there is a clever way to show how circumference and area fit together.

Suppose that you cut a large circular pizza into 8 identical slices. You can reposition those slices to make a shape that is very close to a parallelogram:



How are the circle and the parallelogram related?

Problem 3.4

A The radius of the circular pizza is 6 inches. What are its circumference and area?

B The shape made by re-arranging the pizza slices looks like a parallelogram.

1. Estimate the height and base of the parallelogram. Explain your reasoning.
2. What is the approximate area of the parallelogram?
3. How does the approximate area of the parallelogram compare to the exact area of the circular pizza?

C The connection between the area of the circular pizza and the area of the near-parallelogram formed when the pizza slices are rearranged is only an approximation.

Sara thinks about the parallelogram and says that the area of a circle is $A = \frac{1}{2}(\pi d)(r)$.

Evan thinks about covering the circle with radius squares and says the area is $A = \pi r^2$.

1. Do these formulas give the same area for radius 6 centimeters?
2. Would both formulas work for all values of r? Why or why not?
3. How would the accuracy of the approximation change if you cut the pizza into more slices?

D Suppose that you have 12 meters of fencing and want to make a pen for your pet dog.

Which shape, a square or a circle, would give more area? Explain.

ACE Homework starts on page 58.