Applications

1. Each stack consists of identical cubes. How many cubes are there in each stack?

Stack A

Stack B

2. Bags of sand, cement, or fertilizer are often packed on wooden pallets for shipping.

   a. Estimate the number of bags on the wooden pallet pictured above.

   b. How many loaded pallets in the shape of 1-meter cubes could you load in a semitrailer that has inner dimensions 2 meters wide by 10 meters long by 3 meters high?

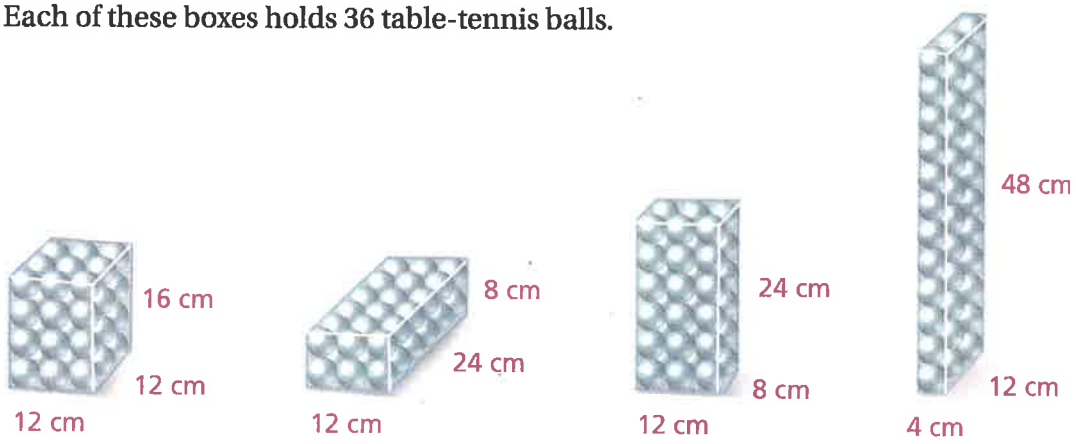
   c. How many loaded pallets in the shape of 1-meter cubes could you ship in a railroad boxcar that has inner dimensions 3 meters wide by 15 meters long by 4 meters high?

3. Suppose you plan to make a box that will hold exactly 40 one-inch cubes.

   a. Give the dimensions of all the possible boxes you can make.

   b. Which of the boxes you described in part (a) has the least surface area? Explain.

4. Each of these boxes holds 36 table-tennis balls.



a. Without calculating, which box has the least surface area? Why?

b. Check your guess by finding the surface area of each box.

5. Liquids, such as juice or milk, and solids, such as rice or cake mix, are often packaged in rectangular boxes. Because the material settles easily into a box of any dimensions, there are many packaging possibilities.

a. What are the volume and surface area of a box that has width 3.5 centimeters, length 10 centimeters, and height 15 centimeters?

b. What are the dimensions of the box that would hold 250 cubic centimeters of juice and have minimum surface area?

6. The box below is a $6 \times 1 \times 2$ arrangement of drink cans. Suppose the dimensions of the box are, in centimeters, $39 \times 13 \times 12.25$.

a. What is the surface area of the box?

b. What is the surface area of a box with the more traditional $4 \times 3 \times 1$ arrangement, which measures, in centimeters, $26 \times 19.5 \times 12.25$?

7. The box below is a $4 \times 3 \times 2$ arrangement of drink cans. Suppose the dimensions of the box are, in centimeters, $26 \times 19.5 \times 24.5$.

a. What is the surface area of the box?

b. What is the surface area of a box with the more traditional $6 \times 4 \times 1$ arrangement, which measures, in centimeters, $39 \times 26 \times 12.25$?

8. Suppose that a company sells laundry soap in boxes that measure 4 inches by 8 inches by 12 inches. The company wants to offer larger economy size boxes.

a. What changes in dimensions would give a box with double the volume?

b. What changes in dimensions would give a box with triple the volume?

c. What changes in dimensions would give a box with half the volume?

d. Given each scale factor below, find the volume of the box similar to the basic box.

i. scale factor of 2

ii. scale factor of 1.5

iii. scale factor of 0.5

Explain how you can find these answers without calculating the separate dimensions of each new box design.

9. You want to make a compost box bigger than the basic 1-2-3 foot size. Your friend says, 'If you double each dimension, you'll be able to double the capacity at only double the cost of the materials to build it.'

a. Is your friend correct about doubling the capacity? Why or why not?

b. Is your friend correct about doubling the cost? Why or why not?

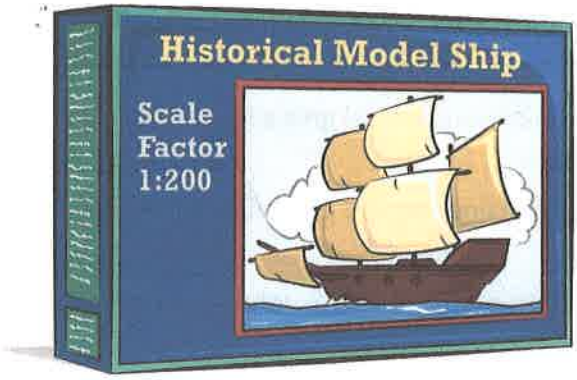
10. The city of Centerville plans to dig a landfill in the shape of a rectangular prism. The landfill is 85 feet deep, 200 feet wide, and 700 feet long.

a. How many cubic feet of garbage will the landfill hold?

b. What information do you need to determine how long the landfill can be used until it is full?

c. The city manager says that an increase of only 10% in each dimension would increase the capacity of the landfill by 33%. Is she correct? Explain.

11. Shaun's hobby is building model airplanes and ships from kits. One of his projects is an historic sailing ship. The actual ship is related to the model by a scale factor of 200.



a. If the length of the model is 25 centimeters, what is the length of the actual ship?

b. If the length of one mast on the actual ship is 30 meters, what is the length of the mast on the model?

c. If the area of the deck on the model is 20 square centimeters, what is the area of the deck on the actual ship?

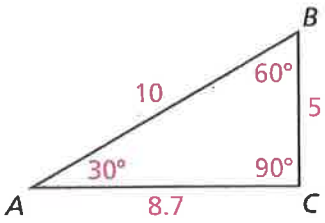
d. If the model ship's interior has volume 100 cubic centimeters, what is the volume of the interior of the actual ship?

Connections

12. Find all the factor pairs and the prime factorizations of each number.

a. 11 **b.** 18 **c.** 42

13. Suppose you enlarge the triangle below by a scale factor of 2.4.



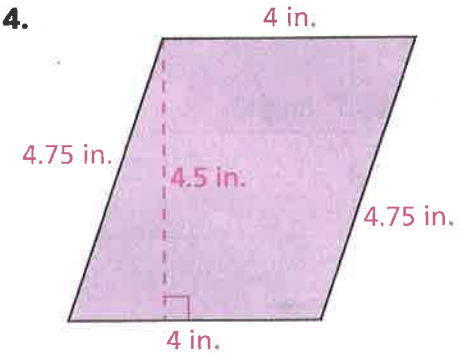
a. What is the perimeter of the image? How can you find this without finding the lengths of each side of the image?

b. What is the area of the image? How can you find this without finding the lengths of each side of the image?

c. What are the measures of the angles in the image? How can you find these measures without measuring the angles in the image?

For Exercises 14–16, find the area and perimeter of each figure. Figures are not drawn to scale.

14.



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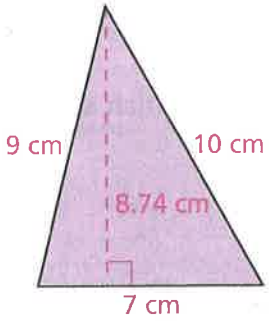

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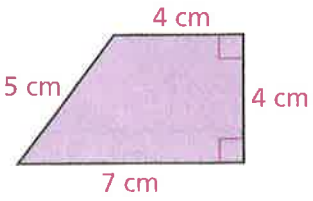
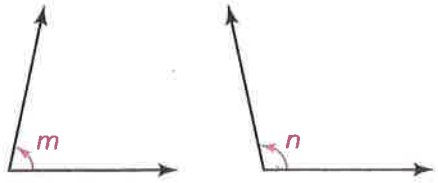
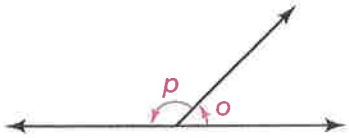


17. Angles m and n below are supplementary angles. Angle m has a measure of 78°. What is the measure of angle n?

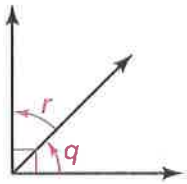


18. Multiple Choice Which pair of angles are complementary angles?

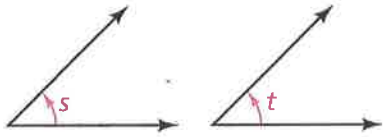
A.



B.



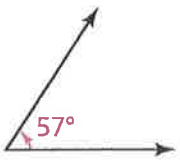
C.



19. Multiple Choice Which angle is supplementary to a 57° angle?

A.

B.



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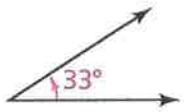
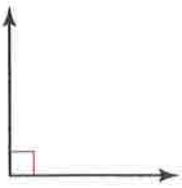
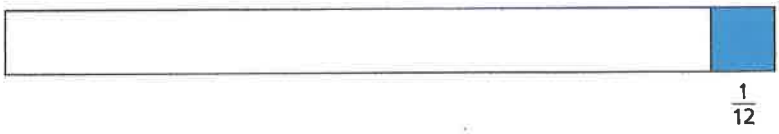


D.



20. Ms. Zhou is making wooden slats for doll beds from a strip of thin board. She cuts $\frac{1}{12}$ of the strip for another project. Bed slats for one doll bed take $\frac{1}{8}$ of a strip.



How many beds can Ms. Zhou make from the remaining board?

21. Calculations of surface areas and volumes often require combining measurements in different units. You can almost always make the conversions by use of rules in the form $y = kx$. Write formulas for these unit conversions.

a. Feet F to inches I (This means to convert any given number of feet to the equivalent number of inches.)

b. Inches I to feet F

c. Centimeters C to meters M

d. Meters M to centimeters C

e. Inches I to centimeters C
Hint: Use 1 inch = 2.54 centimeters.

22. A company that makes compost boxes of various sizes charges \$500 to set up its equipment for production. It then adds a charge per box of \$.15 per square foot based on the surface area of the box.

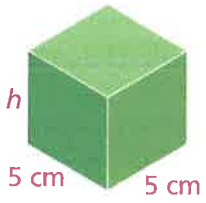
a. The Science Club plans to order compost boxes to sell at Back-to-School Night. The boxes have length 3 feet, width 2 feet, and height 1 foot. What is the surface area of each box (without tops)?

b. How much will it cost the club to buy 100 boxes? 200 boxes? 1,000 boxes?

c. What equation relates cost of a box order C to number of boxes ordered n? Is this a linear relationship? Explain.

d. If the club wants to spend at most \$750 on its box order, how many boxes can it purchase?

23. The square prism below has a volume of 100 cubic centimeters. What is its height?



24. The rectangular prism below has a surface area of 158 square centimeters. What is its height?

25. Four friends share $\frac{3}{5}$ of a rectangular pizza.

a. What fraction of the pizza did each receive?

b. Draw a diagram to illustrate your answer.

26. Mr. Bouck is making snack bars for a camping trip. The recipe calls for $\frac{3}{8}$ stick of butter. Mr. Bouck has $3\frac{1}{2}$ sticks on hand.

a. How many batches of the recipe can Mr. Bouck make?

b. Draw a diagram to illustrate your answer.

27. Taye plans to plant an herb garden in a glass tank. A scoop of dirt fills 0.15 of the volume of the tank. Taye needs to put in dirt equal to 65% of the volume. How many scoops of dirt does he need?

28. **a.** The dimensions of a recreation center floor are 150 feet by 90 feet. The walls of the recreation center are 20 feet high. A gallon of paint will cover about 400 square feet. About how much paint is needed to paint the walls of the recreation center?

b. If a gallon of paint costs $25.50, about how much will it cost to buy the paint for the recreation center walls?

Extensions

29. The number cubes used in many games of chance have the special property that the numbers on opposite faces always add to 7.

a. Draw a net that you can assemble into a cube.

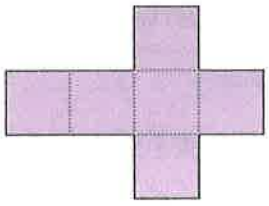
b. Enter the numbers 1, 2, 3, 4, 5, and 6 on the net in a way that guarantees the number cube made from that net meets the condition of opposite faces adding to 7.

c. Is there more than one way to enter the numbers on your particular net? Why or why not?

30. The net for any 1-inch cube will have six 1-inch squares arranged in some pattern.

a. Considering only area, how many nets for 1-inch cubes should you be able to cut from a rectangle that is 9 inches long and 4 inches wide?

b. Can you find a way to actually cut that number of cube nets in the pattern shown below from a 9×4 rectangle? Why or why not?



31. Each expression below will help you to find either the volume or surface area of one of the boxes pictured. Simplify each expression. Decide whether you have found a volume or a surface area, and for which box.

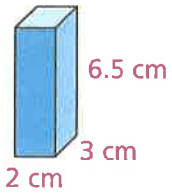
a. $2 \times (3.5 \times 5.7) + 2 \times (5.7 \times 12) + 2 \times (3.5 \times 12)$

b. $6\frac{1}{4} \times 6$

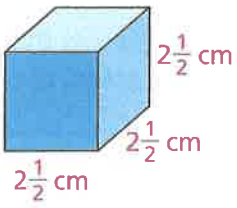
c. $6 \times 6\frac{1}{2}$

d. $2\frac{1}{3} \times 2\frac{2}{5} \times 5$

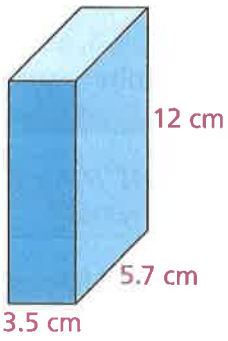
Box A



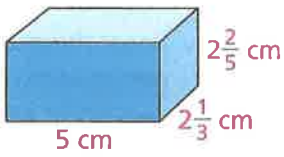
Box B



Box C



Box D



32. **a.** What is the formula for the surface area of a rectangular prism with length ℓ, width w, and height h?

b. If you stretch or shrink a rectangular prism with dimensions ℓ, w, and h by a scale factor of f, what will be the surface area of the new rectangular prism?

c. Explain how the Distributive Property shows the following:

$$\text{Area}_{\text{new}} = f^2 \cdot \text{Area}_{\text{original}}$$

33. **a.** What is the formula for the volume V of a rectangular prism with length ℓ, width w, and height h?

b. If you stretch or shrink a rectangular prism with dimensions ℓ, w, and h by a scale factor of f, what will be the volume V of the new rectangular prism?

c. Explain how the expression in part (b) shows the following:

$$\text{Volume}_{\text{new}} = f^3 \cdot \text{Volume}_{\text{original}}$$