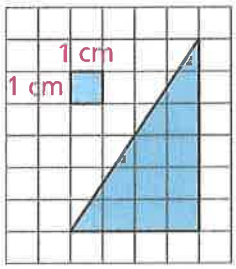
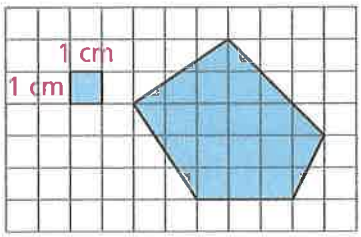
Applications

1. Suppose that the polygons below were drawn on centimeter grid paper. How many 1-centimeter cubes (some cut in pieces) would it take to cover each polygon?

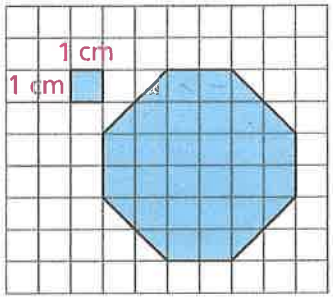
a.



b.

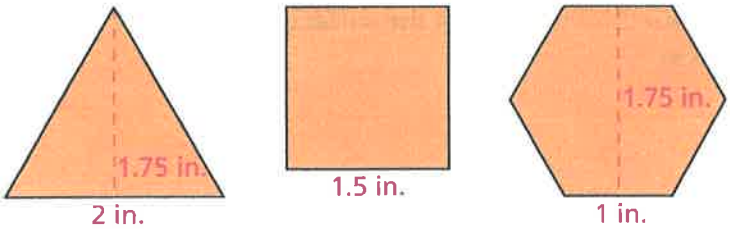


c.



2. Using the method from Problem 2.1, Darius and Mariana made paper prisms from 4 inch-by-6 inch pieces of paper. One has an equilateral triangle for its base, another has a square base, and the third has a regular hexagon for its base. The height of each prism is 4 inches.

To find the areas of the base and top polygons, they traced and measured those figures, as accurately as they could, to get the data shown below. The figures are not drawn to scale.



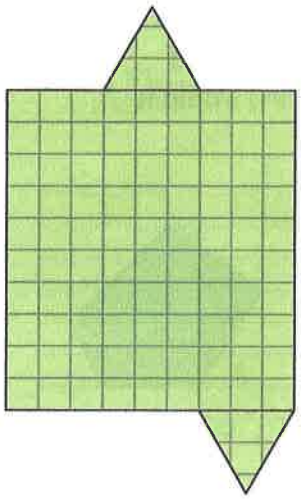
a. What are the areas of the sides of each prism?

b. What are the perimeters of the bases (and tops) of the each prism?

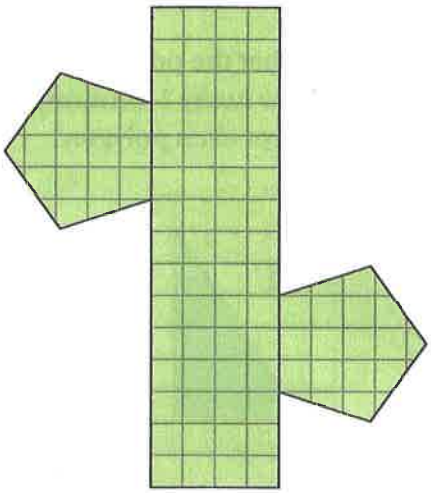
c. What are the areas of the bases (and tops) of each prism?

For Exercises 3–5, use a copy of each of the figures on centimeter grid paper. Cut out each figure and tape it together to make a prism. Use the resulting prisms to answer parts (a) and (b) for each figure.

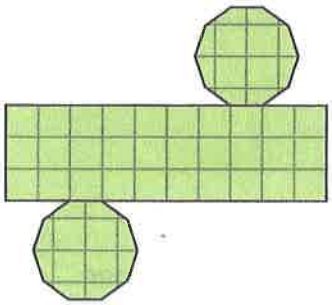
3.



4.



5.



a. What is the surface area of each prism? Explain your reasoning.

b. What is the volume of each prism? Explain your reasoning.

6. a. Describe how to find the surface area of any prism.

b. Compare your methods for finding the surface area of any prism with your method for finding the surface area of a rectangular prism.

7. **a.** Describe how to find the volume of any prism.

b. Compare your methods for finding the volume of any prism with your method for finding the volume of a rectangular prism.

8. Suppose that the figures shown in Exercise 1 are the bases of prisms that are 10 centimeters tall (a triangular prism, a pentagonal prism, and an octagonal prism). What is the volume of each prism?

9. Use your answers to Exercise 2 to find the volumes of the prisms that Darius and Mariana created.

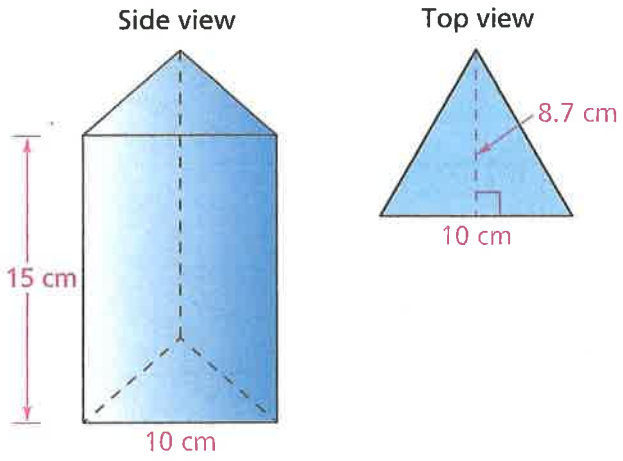
a. The triangular prism

b. The square prism

c. The hexagonal prism

d. Do the results in parts (a)–(c) fit the same pattern relating areas and volumes of the three prisms as your work in Problem 2.1? Why or why not?

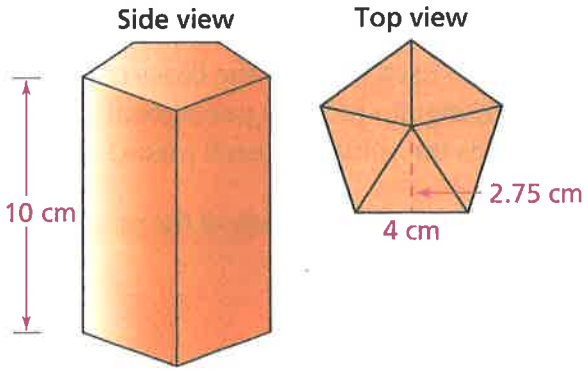
10. Side and top views of a prism whose base and top are equilateral triangles are shown below.



a. What is the surface area of the prism?

b. What is the volume of the prism?

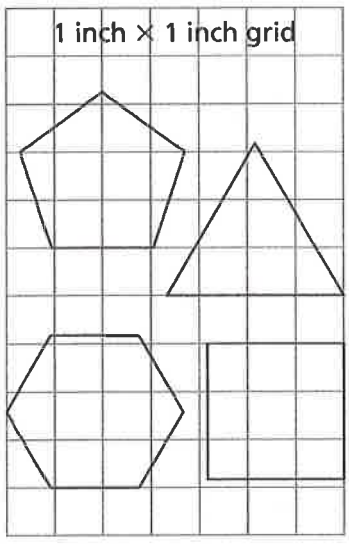
11. The sketch below shows side and top views of a prism with base and top that are regular pentagons.



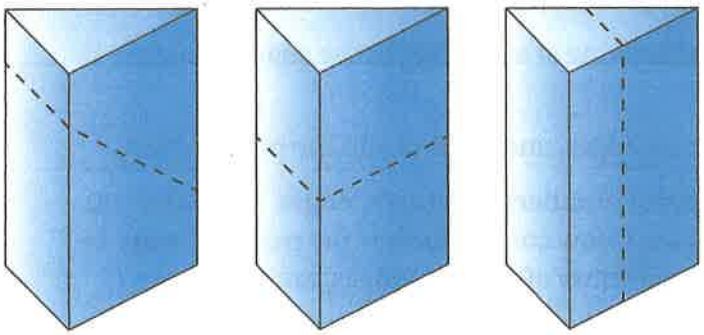
a. What is the surface area of the prism?

b. What is the volume of the prism?

12. For Problem 2.1, Sheryl made paper prisms that were all 8.5 inches high. She traced the polygon bases on 1-inch grid paper to give a picture like the one shown below. Estimate the volume of each prism.



13. Suppose that you slice three triangular prisms as shown below.

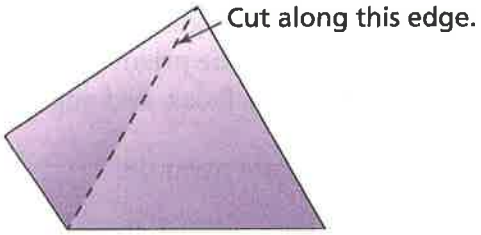


Slant cut | Horizontal cut | Vertical cut

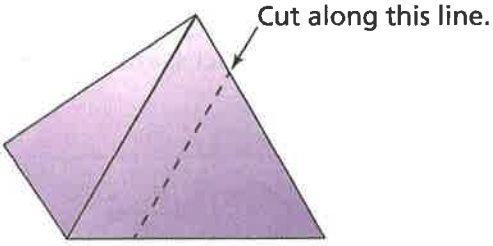
For each cut, answer the following questions.

- How many faces will there be on each figure created by the slice?
- What different polygons appear as faces on each figure?

14. **a.** What kinds of figures will result if you cut a square pyramid along one edge as shown below? Describe the number and shapes of the faces of the new figures formed.



b. What kinds of figures will result if you cut into a square pyramid as shown below? Describe the number and shapes of the faces of the new figures formed.



Connections

15. Suppose that you are looking at a building in the shape of a triangular prism.

a. How many vertices, edges, and faces would the prism have?

b. What are the greatest numbers of vertices, edges, and faces you would be able to see? How could you view the prism in order to see the maximum number of vertices, edges, and faces?

c. What are the smallest numbers of vertices, edges, and faces you would be able to see? How could you view the prism in order to see only the minimum number of vertices, faces, and edges?

16. Suppose that you are looking at a building in the shape of a prism with regular pentagons for base and top.

a. How many vertices, edges, and faces would the prism have?

b. What are the greatest numbers of vertices, edges, and faces you would be able to see? How could you view the prism in order to see the maximum number of vertices, edges, and faces?

c. What are the smallest numbers of vertices, edges, and faces you would be able to see? How could you view the prism in order to see only the minimum number of vertices, faces, and edges?

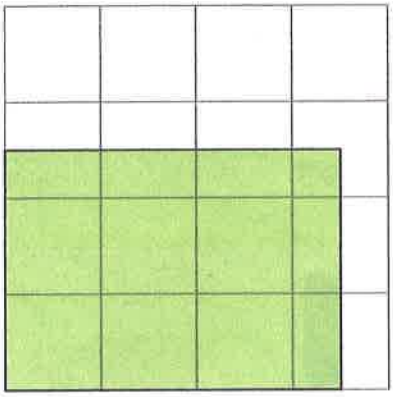
17. Suppose that you are looking at a large square pyramid.

a. How many vertices, edges, and faces would the pyramid have?

b. What are the greatest numbers of vertices, edges, and faces you would be able to see? How could you view the prism in order to see the maximum number of vertices, edges, and faces?

c. What are the smallest numbers of vertices, edges, and faces you would be able to see? How could you view the prism in order to see only the minimum number of vertices, faces, and edges?

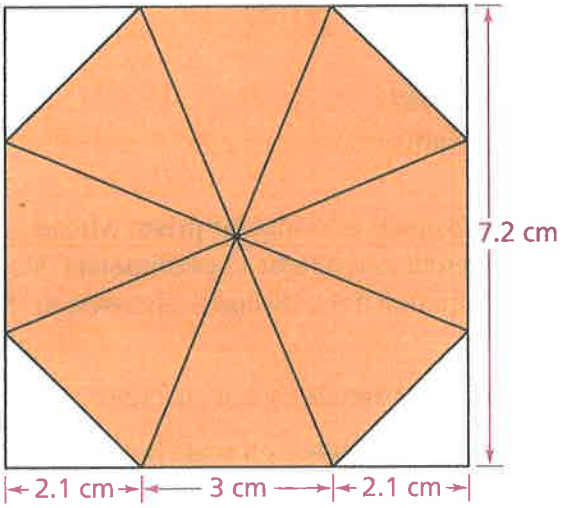
18. The figure below shows a rectangle drawn on a grid of identical squares.



a. What is $3\frac{1}{2} \times 2\frac{1}{2}$? How does the diagram confirm your result?

b. What is 3.5×2.5? How does the diagram confirm your result?

19. The regular octagon in the figure below is enclosed in a square.

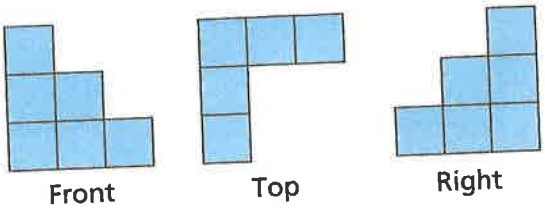


a. Does the calculation $8(0.5 \times 3 \times 3.6)$ give the area of the octagon? Why or why not?

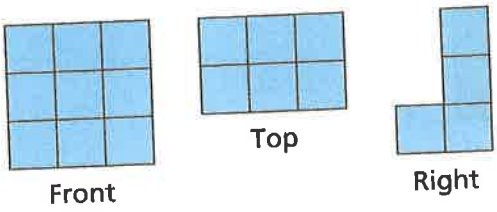
b. Does the calculation $(7.2 \times 7.2) - 4(0.5 \times 2.1 \times 2.1)$ give the area of the octagon? Why or why not?

For Exercises 20–22, use the three given views of a three-dimensional figure to sketch the figure itself. Then, find its volume.

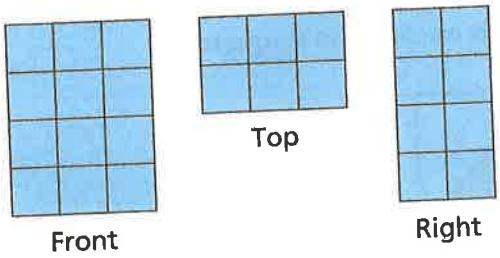
20.



21.



22.



23. Suppose that the design of a package is a triangular prism whose base has area 12 square centimeters and height 15 centimeters. How will the volume of the package change if the design is changed in the following ways?

a. The base is stretched to a similar triangle by a scale factor of 1.5.

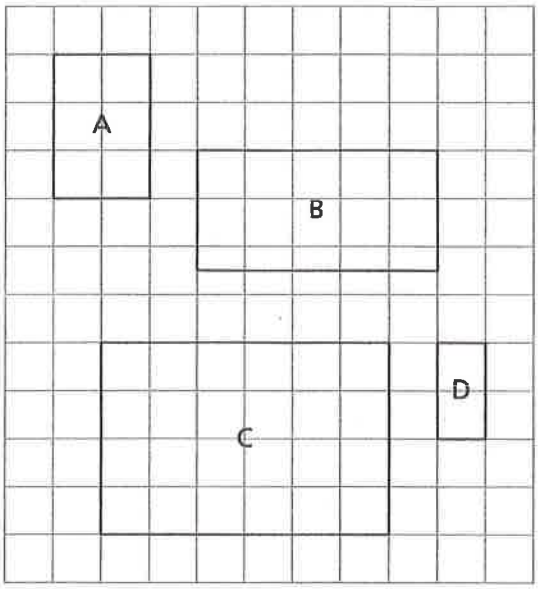
b. The base is stretched to a similar triangle by a scale factor of 3.

c. The base is shrunk to a similar triangle by a scale factor of 0.6.

d. The height of the prism is increased by a factor of 3.

e. The height of the prism is increased to 18 centimeters.

24. The figure below shows four rectangles on a grid of squares.



a. Which pairs of rectangles are similar? Explain.

b. Suppose that a fifth rectangle has length 12 and is similar to rectangle D in the figure above. What must be the width of the fifth rectangle?

25. Suppose you are standing inside a large rectangular prism, such as your school's gymnasium.

a. What are the greatest numbers of vertices, edges, and faces you could see by moving your eyes, but not moving your head? Where would you stand?

b. What are the smallest numbers of vertices, edges, and faces you could see by moving your eyes, but not moving your head? Where would you stand?

26. Kwan has two prism-shaped containers. One has a volume of $3\frac{3}{4}$ cubic feet. The other has a volume of $\frac{1}{3}$ cubic foot.

a. How many of the smaller container will it take to fill the larger?

b. What operation(s) did you use to find the answer? Explain.

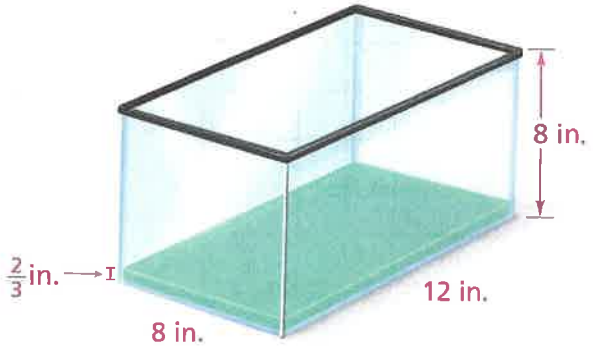
27. Antonia has two prism-shaped containers. One has a volume of $2\frac{2}{5}$ cubic feet. The other has a volume of $\frac{2}{3}$ cubic foot.

a. How many of the smaller container will it take to fill the larger?

b. What operation(s) did you use to find the answer? Explain.

28. The diagram below shows a fish tank after a container of water is poured into it.

Tank A



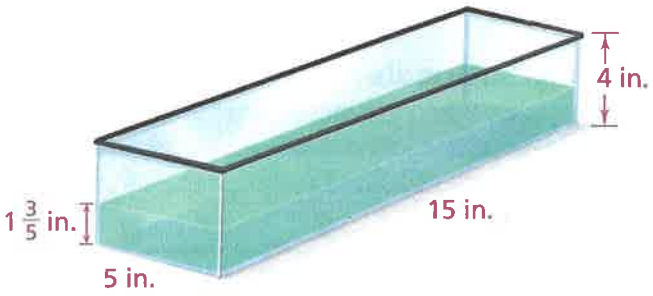
a. Using the same container, how many containers of water are needed to fill the tank?

b. What fraction of the tank does the container fill?

c. A different container holds $12\frac{3}{4}$ cubic inches of water. How many of these containers are needed to fill the tank?

29. The diagram below shows a fish tank after a container of water is poured into it.

Tank B



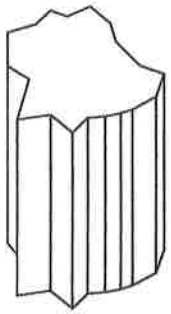
a. Using the same container, how many containers of water are needed to fill the tank?

b. What fraction of the tank does the container fill?

c. A different container holds $4\frac{4}{9}$ cubic inches of water. How many of these containers are needed to fill the tank?

Extensions

30. The drawing below shows a prism with an odd-shaped but congruent top and bottom. The top and bottom each have an area of 10 square centimeters. The height of the prism is 4 centimeters.



a. What is the volume of the prism? Explain your reasoning.

b. Is your estimate for the volume more than, less than, or equal to the exact volume? Explain.

31. Leonhard Euler made contributions in many areas of mathematics. One of his most widely known discoveries is a formula relating the vertices, edges, and faces of solid figures like prisms and pyramids.

a. Record data about figures you have studied in a table like the one below.

Euler's Formula

Figure	Vertices	Edges	Faces
Rectangular Prism	8	12	6
Triangular Prism			
Pentagonal Prism			
Hexagonal Prism			
Triangular Pyramid			
Square Pyramid			

b. Study the data to see if you can discover Euler's Formula relating the number of vertices V, edges E, and faces F in figures such as the prisms and pyramids you have studied.

c. Test your ideas by counting vertices, faces, and edges on other solid figures that have polygonal faces.