Applications

1. Find the values of each pair of expressions.

   a. $-12 + (-4 + 9)$ and $[-12 + (-4)] + 9$

   b. $(14 - 20) - 8$ and $14 - (20 - 8)$

   c. $[14 + (-20)] + (-8)$ and $14 + [-20 + (-8)]$

   d. $-1 - [-1 + (-1)]$ and $[-1 - (-1)] + (-1)$

   e. Which cases lead to expressions with different results? Explain.

For Exercises 2–7, find the value of each expression.

2. $(5 - 3) \div (-2) \cdot (-1)$

3. $2 + (-3) \cdot 4 - (-5)$

4. $4 \cdot 2 \cdot (-3) + (-10) \div 5$

5. $-3 \cdot [2 + (-10)] - 2^2$

6. $(4 - 20) \div 2^2 - 5 \cdot (-2)$

7. $10 - [50 \div (-2 \cdot 25) - 7] \cdot 2^2$

For Exercises 8–11, rewrite each expression in an equivalent form to show a simpler way to do the arithmetic. Explain how you know the two results are equal without doing any calculations.

8. $(-150 + 270) + 30$

9. $(43 \cdot 120) + [43 \cdot (-20)]$

10. $23 + (-75) + 14 + (-23) - (-75)$

11. $[0.8 \cdot (-23)] + [0.8 \cdot (-7)]$

12. Without doing any calculations, determine whether each number sentence is true. Explain. Then check your answer.

   a. $50 \cdot 432 = (50 \cdot 400) + (50 \cdot 32)$

   b. $50 \cdot 368 = (50 \cdot 400) - (50 \cdot 32)$

   c. $-50 \cdot 998 = [-50 \cdot (-1{,}000)] + [-50 \cdot 2]$

   d. $-50 + (400 \cdot 32) = (-50 + 400) \cdot (-50 + 32)$

   e. $(-70 \cdot 20) + (-50 \cdot 20) = (-120 \cdot 20)$

   f. $6 \cdot 17 = 6 \cdot 20 - 6 \cdot 3$

For each part, use the Distributive Property to write an equivalent expression.

13. $-2 \cdot [5 + (-8)]$

14. $(-3 \cdot 2) - [-3 \cdot (-12)]$

15. $x \cdot (-3 + 5)$

16. $-7x + 4x$

17. $2x \cdot [2 - (-4)]$

18. $x - 3x$

19. A grocery store receipt shows 5% state tax due on laundry detergent and a flower bouquet. Does it matter whether the tax is calculated on each separate item or the total cost? Explain.

Connections

For Exercises 20–37, find the sum, difference, product, or quotient.

20. $3 \cdot 12$

21. $3 \cdot (-12)$

22. $-3 \div (-12)$

23. $-10 \cdot (-11)$

24. $-10 + 11$

25. $10 - 11$

26. $-24 - (-12)$

27. $\frac{-24}{-12}$

28. $-18 \div 6$

29. $50 \cdot 70$

30. $50 \cdot (-70)$

31. $2{,}200 \div (-22)$

32. $-50 \cdot (-120)$

33. $-139 + 899$

34. $5{,}600 - 7{,}800$

35. $-4{,}400 - (-1{,}200)$

36. $\frac{-9{,}900}{-99}$

37. $-580 + (-320)$

38. When using negative numbers and exponents, you sometimes need parentheses to make it clear what you are multiplying.

You can think of -5^4 as "the opposite of 5^4" or
$-(5^4) = -(5 \cdot 5 \cdot 5 \cdot 5) = -625$

You can think of $(-5)^4$ as "negative five to the fourth power" or
$(-5)^4 = -5 \cdot (-5) \cdot (-5) \cdot (-5) = 625$

Indicate whether the following expressions will be negative or positive. Explain your answers.

a. -3^2 **b.** $(-6)^3$ **c.** $(-4)^4$

d. -1^6 **e.** $(-3)^4$ **f.** -2^3

39. This list shows the yards gained and lost during the first several plays of a football game:

8, 4, 3, 7, −15, 20, 5, −12, 32, 1

Write an expression that shows how to compute the team's average gain or loss per play. Then compute the average.

40. Complete each number sentence.

a. $-34 + (-15) = \blacksquare$

b. $-12 \cdot (-23) = \blacksquare$

c. $-532 \div \blacksquare = -7$

d. $-777 - \blacksquare = -740$

e. Write a fact family for part (a).

f. Write a fact family for part (b).

For Exercises 41–44, write a related fact. Use it to find the value of n that makes the sentence true.

41. $n - (-5) = 35$ **42.** $4 + n = -43$

43. $-2n = -16$ **44.** $\frac{n}{4} = -32$

45. Insert parentheses (or brackets) in each expression if needed to make the equation true.

a. $1 + (-3) \cdot (-4) = 8$

b. $1 + (-3) \cdot (-4) = 13$

c. $-6 \div (-2) + (-4) = 1$

d. $-6 \div (-2) + (-4) = -1$

e. $-4 \cdot 2 - 10 = -18$

f. $-4 \cdot 2 - 10 = 32$

46. Multiple Choice Which set of numbers is in order from least to greatest?

A. 31.4, −14.2, −55, 75, −0.05, 0.5, 3.140

B. $\frac{2}{5}, \frac{-3}{5}, \frac{8}{7}, \frac{-9}{8}, \frac{-3}{2}, \frac{5}{3}$

C. −0.2, −0.5, 0.75, 0.6, −1, 1.5

D. None of these

47. Find the absolute values of the numbers for each set in Exercise 46. Write them in order from least to greatest.

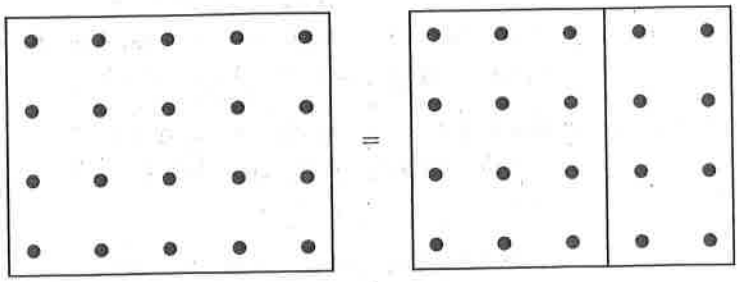
For Exercises 48–50, decide whether each statement is correct, and explain your answer.

48. $|-2 + 3| = |-2| + |3|$

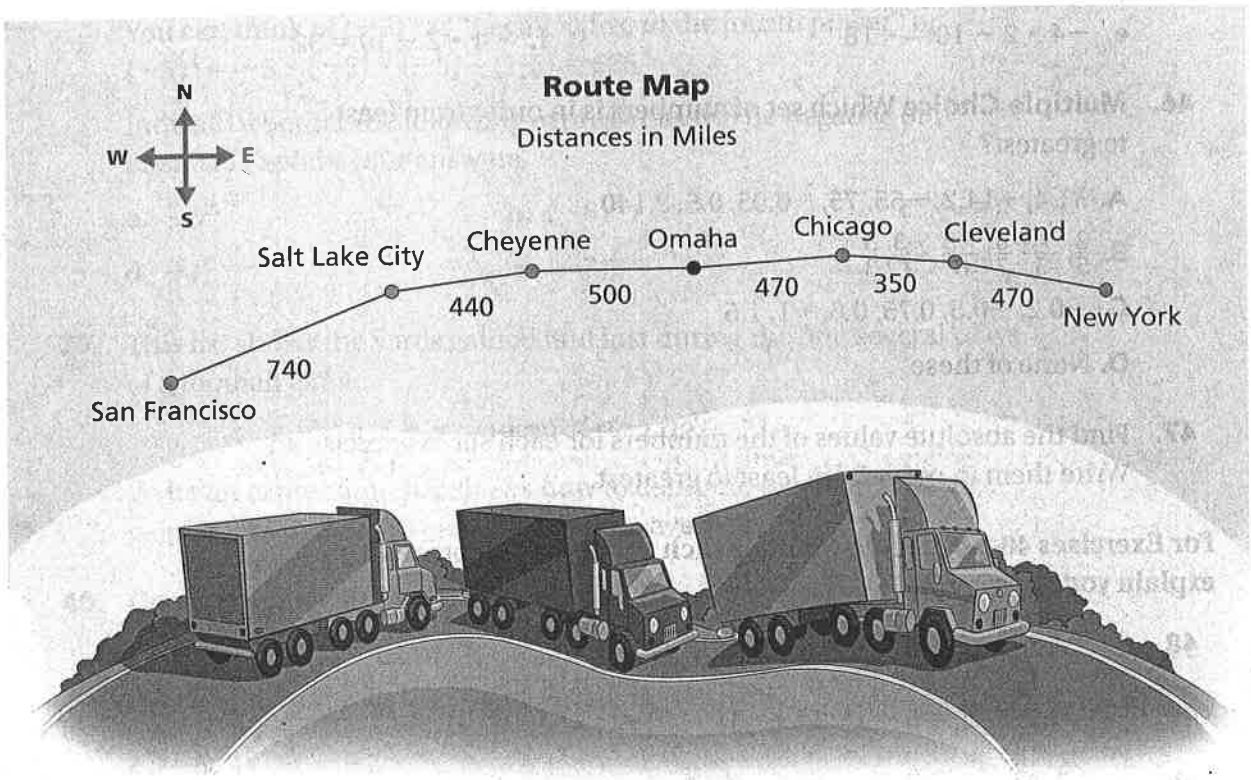
49. $5 - |-2 + 3| = 5 - |-2| + |3|$

50. $|-2 - 3| = |-2| + |-3|$

51. You can use dot patterns to illustrate the distributive properties for operations on whole numbers. Write a number sentence to represent the pair of dot patterns.



52. A trucking company carries freight along a highway from New York City to San Francisco. Its home base is in Omaha, Nebraska, which is about halfway between the two cities. Truckers average about 50 miles per hour on this route.



Make a number line to represent this truck route. Put Omaha at 0. Use positive numbers for cities east of Omaha and negative numbers for cities west of Omaha. Then write number sentences to answer each question.

a. A truck leaves Omaha heading east and travels for 7 hours. About how far does the truck go? Where on the number line does it stop?

b. A truck leaves Omaha heading west and travels for 4.5 hours. About how far does the truck go? Where on the number line does it stop?

c. A truck heading east arrives in Omaha. About where on the number line was the truck 12 hours earlier?

d. A truck heading west arrives in Omaha. About where on the number line was the truck 11 hours earlier?

Extensions

Copy each pair of expressions in Exercises 53–57. Insert $<$ or $>$ to make a true statement.

53. $-23 \blacksquare -45$

54. $-23 + 10 \blacksquare -45 + 10$

55. $-23 - 10 \blacksquare -45 - 10$

56. $-23 \cdot 10 \blacksquare -45 \cdot 10$

57. $-23 \cdot (-10) \blacksquare -45 \cdot (-10)$

For Exercises 58–60, refer to your results in Exercises 53–57. Complete each statement. Test your ideas with other numerical cases, or develop another kind of explanation, perhaps using chip board or number line ideas.

58. If $a > b$, then $a + c \blacksquare b + c$.

59. If $a > b$, then $a - c \blacksquare b - c$.

60. If $a > b$, then $a \cdot c \blacksquare b \cdot c$.

For Exercises 61–63, find the value for n that makes the sentence true.

61. $n - (-24) = 12$

62. $2.5n = -10$

63. $2.5n + (-3) = -13$

64. Complete each pair of calculations.

a. $12 \div (-8 + 4) = \blacksquare$ $\qquad$ $[12 \div (-8)] + (12 \div 4) = \blacksquare$

b. $-12 \div [-5 - (-3)] = \blacksquare$ $\qquad$ $[-12 \div (-5)] - [-12 \div (-3)] = \blacksquare$

c. $(-2 - 6) \div 4 = \blacksquare$ $\qquad$ $(-2 \div 4) - (6 \div 4) = \blacksquare$

d. $(5 + 6) \div 3 = \blacksquare$ $\qquad$ $(5 \div 3) + (6 \div 3) = \blacksquare$

e. What can you conclude from parts (a)–(d) about the Distributive Property?

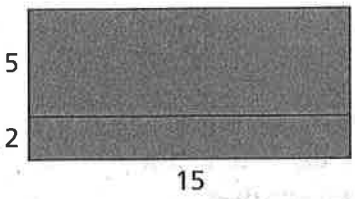
65. When you find the mean (average) of two numbers, you add them together and divide by 2.

a. Does the order in which you do the operations matter? Give examples.

b. Does multiplication distribute over the averaging operation? That is, will a number a times the average of two numbers, x and y, give the same result as the average of ax and ay? Give examples.

For Exercises 66–69, write equivalent expressions to show two different ways to find the area of each rectangle. Use the ideas of the Distributive Property.

66.



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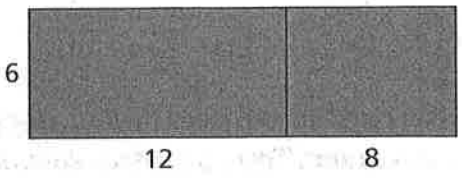

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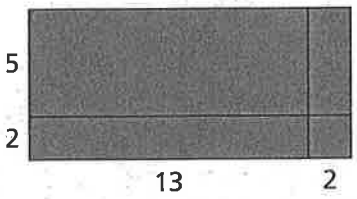

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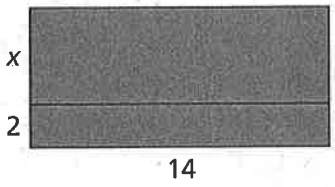

For Exercises 70–73, draw and label the edges and areas of a rectangle to illustrate each pair of equivalent expressions.

70. $(3 + 2) \cdot 12 = 3 \cdot 12 + 2 \cdot 12$

71. $9 \cdot 3 + 9 \cdot 5 = 9 \cdot (3 + 5)$

72. $x \cdot (5 + 9) = 5x + 9x$

73. $2 \cdot (x + 8) = 2x + 16$