3.3 Division of Rational Numbers

You know that there is a relationship between addition and subtraction facts. A similar relationship exists between multiplication and division. For any multiplication fact, you can write another multiplication fact and two different related division facts. Here are three examples of rational-number fact families.

Example 1	Example 2	Example 3
$5 \cdot 3 = 15$	$6 \cdot (-3) = -18$	$4.5 \cdot (-2) = -9$
$3 \cdot 5 = 15$	$-3 \cdot 6 = -18$	$-2 \cdot 4.5 = -9$
$15 \div 3 = 5$ or $\frac{15}{3} = 5$	$-18 \div (-3) = 6$ or $\frac{-18}{-3} = 6$	$-9 \div (-2) = 4.5$ or $\frac{-9}{-2} = 4.5$
$15 \div 5 = 3$ or $\frac{15}{5} = 3$	$-18 \div 6 = -3$ or $\frac{-18}{6} = -3$	$-9 \div 4.5 = -2$ or $\frac{-9}{4.5} = -2$

Recall that a rational number can be written as $\frac{a}{b}$ where a and b are integers and b is not zero. Fact families help to clarify why division by zero is impossible. If $\frac{15}{0} = a$, then $a \cdot 0 = 15$ and $\frac{15}{0} = a$ are in the same fact family.

- How does a fact family show that $\frac{15}{0} = a$ cannot be a true statement for any value of a?

Problem 3.3

A Use what you know about fact families and multiplication to rewrite, if necessary, and find the missing value. Then find the missing value.

1. $-6 \times (-13) = \square$
2. $6 \times (-13) = \square$
3. $\square \times (-9) = 108$
4. $8 \times \square = -48$

B The team in Problem 3.1 runs another relay. Write division sentences that express your answers to the questions below.

1. Dwayne goes from 0 to 15 meters in 5 seconds. At what rate (meters per second) does he run?
2. Pascal reaches -12 meters only 3 seconds after passing 0. At what rate does he run?
3. Aurelia passes 0 running to the right at a rate of 5 meters per second. When did she leave the point -50? When did she leave the point -24?
4. Tori wants to reach the point -40, running to the left at 8 meters per second. How long will it take her from the time she passes 0?

C 1. What do the examples in each group have in common?

Group 1	Group 2	Group 3
$12 \div 3$	$12 \div (-3)$	$-12 \div (-3)$
$4.5 \div 9$	$-4.5 \div 9$	$-4.5 \div (-9)$
$2\frac{1}{4} \div \frac{1}{2}$	$2\frac{1}{4} \div \left(-\frac{1}{2}\right)$	$-2\frac{1}{4} \div \left(-\frac{1}{2}\right)$

2. Find the quotients in each group above.
3. Write and solve two additional problems for each group.
4. Describe an algorithm for dividing rational numbers.

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Problem 3.3 continued

D **1.** Find the quotients in each group below. Is division commutative?

$$-2 \div 3 \text{ and } 3 \div (-2)$$

$$-12 \div (-4) \text{ and } -4 \div (-12)$$

$$16 \div 8 \text{ and } 8 \div 16$$

2. Give two other examples to support your answer to part (1).

E **1.** Zero is the additive identity for addition. For example, $0 + a = a$, where a is a rational number. Explain in words what this means. Provide an example.

2. Is there a *multiplicative identity* n such that $a \cdot n = a$ for any rational number a? Explain.

F **1.** Each rational number has an additive inverse. For example, $a + (-a) = 0$, where a is a rational number. Explain in words what this means. Provide an example.

2. Is there a *multiplicative inverse* b, such that $a \times b = 1$, for each rational number a? Explain.

G Use properties of multiplication and division to find each value. State which properties you use.

1. $\dfrac{\frac{5}{4} \times 7}{\frac{5}{4}}$

2. $\frac{3}{5}\left(\frac{5}{3}\right)$

3. $0.2 \times 3 \times \frac{1}{0.2}$

4. $\dfrac{1.3 \times 8.2}{1.3}$

5. $\frac{4}{5}\left(\frac{10}{3} + \frac{15}{6}\right)$

6. $-\frac{3}{5}\left(-\frac{8}{21}\right)\left(-\frac{5}{3}\right)$

7. $1.6 \times \frac{5}{8} - 2.4 \times \frac{5}{8}$

8. $\dfrac{-2.4 \times \frac{4}{7}}{\frac{4}{7}}$

Problem 3.3 continued

H Recall that some fractions have decimals that terminate. For example, $\frac{3}{4} = 0.75$. Other fractions have decimals that repeat. For example, $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$. The 3 repeats.

1. State whether each fraction will *terminate* or *repeat*. Then write each fraction as a decimal.

   a. $\frac{2}{5}$ **b.** $\frac{3}{8}$ **c.** $\frac{-5}{6}$

   d. $\frac{35}{10}$ **e.** $\frac{8}{-9}$ **f.** $\frac{-3}{-11}$

2. List two other fractions that will terminate and two that will repeat. Give their decimal representations.

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Note on Notation You know that a rational number is any number that you can write in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. When a rational number is negative, the negative sign can be associated with the numerator, the denominator, or the entire fraction. For positive integers a and b,

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

For example, suppose $a = 6$ and $b = 2$.

$$\frac{-6}{2} = \frac{6}{-2} = -\frac{6}{2} = -3$$