2.2 Extending Subtraction to Rational Numbers

In Problem 2.1, you explored some important properties of rational numbers. You found that the Commutative Property is true for addition of rational numbers.

You also found that the sum of an integer and its opposite is 0.

$$50 + {}^{-}50 = 0 \qquad {}^{-}17 + 17 = 0$$

Numbers such as 50 and -50 are **additive inverses** of each other. Their sum is 0. Zero is the **additive identity** for rational numbers. This means that zero added to a number does not change the value of the number.

$$^{-}7 + 0 = {}^{-}7 \qquad \frac{1}{2} + 0 = \frac{1}{2}$$

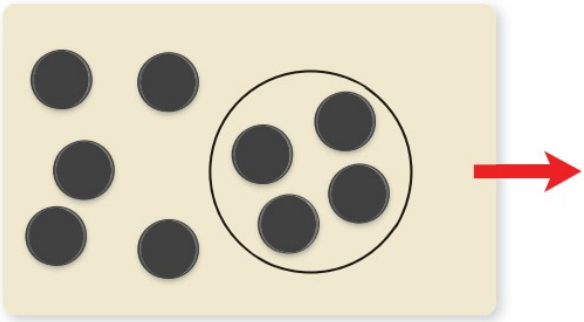
These properties will be useful as you explore subtraction problems with rational numbers.

One way to think about subtraction problems is to take away objects from a set, as in this example:

> Kim had 9 DVDs. She sold 4 at a yard sale. She now has $9 - 4 = 5$ of those DVDs left.

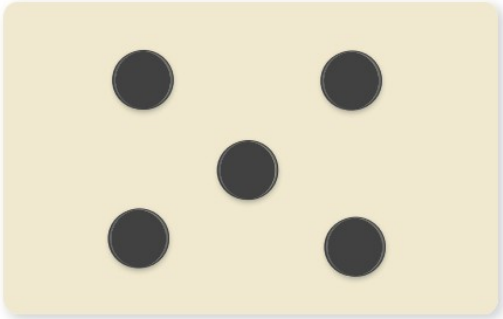
One way to represent this situation is to use a chip board

9 − 4 = 5



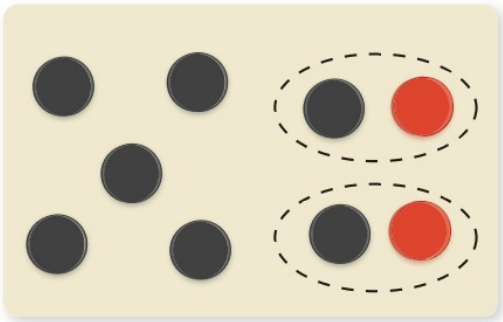
Here is another example:

> Otis earned $5 raking leaves. He wants to buy a used bike that costs $7. His older sister puts 5 black chips on the table to represent the money Otis has.



- What is the value of Otis's board?

Otis's sister asks, "How much more money do you need?" Otis replies, "I could find out by taking away $7. But I can't take away $7 because there aren't seven black chips on the board!" His sister adds two black chips and two red chips.



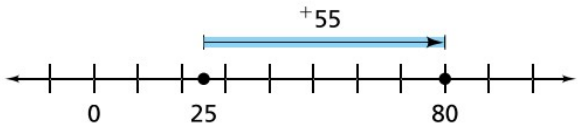
- Is the value of the board the same with the new chips added? Explain.
- How does this help Otis find how much more he needs?

You can also use subtraction to find the distance between two points:

> The Arroyo family just passed mile 25 on the highway. They need to get to the exit at mile 80.

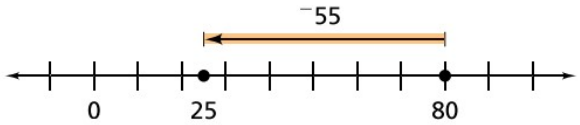
- How many more miles do they have to drive?

You can use a number line to show this difference.



The number line above shows that they have to travel $80 - 25 = 55$ more miles. The arrow on the number line points in the direction that the Arroyos are traveling. They are traveling in a positive direction, from lesser values to greater values.

Suppose the Arroyos drive back from mile 80 to mile 25. They would travel the same distance as before. However, they would travel in the opposite direction.



The number line above represents the Arroyos' distance as $25 - 80 = {}^{-}55$ miles. In this case, the arrow on the number line points to the left and has a label of $^{-}55$. Their distance is 55, but their direction is negative.

In some situations, such as driving, it makes more sense to describe an overall distance without including the direction. You can find the Arroyos' overall distance by taking the **absolute value** of the difference between the two points on the number line.

You can write two absolute value expressions to represent the distance between 25 and 80:

$$|25 - 80| \text{ and } |80 - 25|$$

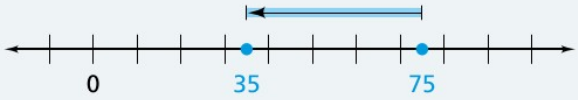
You can evaluate these two expressions to show that the distance between the points 25 and 80 on a number line is 55.

$$|25 - 80| = |{}^{-}55| = 55 \text{ and } |80 - 25| = |55| = 55$$

How can you predict whether the difference of two integers is 0, positive, or negative?

Problem 2.2

A Benjamin takes $75 from his savings. He goes shopping for school supplies and has $35 left when he is done. To figure out how much he has spent, he draws the following number line:



1. How much has he spent?
2. How should Benjamin label what he spent to show that this is money that he no longer has?

B During a game of Math Fever the Super Brains have a score of $^{-}500$ points. Earlier in the game, they incorrectly answered a question for $^{-}150$ points. However, the moderator later determined that the question was unfair. So $^{-}150$ points are taken away from their score.

1. Will subtracting $^{-}150$ points increase or decrease the Super Brains' score? Explain your reasoning.
2. What is the Super Brains' score after $^{-}150$ points are removed?
3. Write a number sentence to represent this situation, and show it on a number line.

C Use chip models or number line models to help solve the following.

1. Find the differences in each group given below.

Group 1
$^{+}12 - {}^{+}8$
$^{-}5 - {}^{-}7$
$^{-}4 - {}^{-}2$
$^{+}2 - {}^{+}4$

Group 2
$^{+}12 - {}^{-}8$
$^{-}5 - {}^{+}7$
$^{-}4 - {}^{+}2$
$^{+}2 - {}^{-}4$

2. What do the examples in each group have in common?
3. Write two new problems that belong to each group.
4. Describe an algorithm for subtracting integers in each group.

continued on the next page >

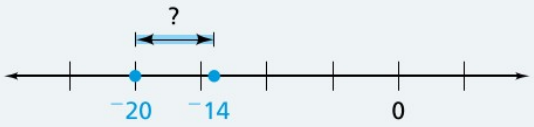
Problem 2.2 continued

D Apply the algorithm you developed on these rational number problems.

1. $^{-}1 - {}^{+}3$
2. $^{-}1 - \frac{^{+}3}{4}$
3. $^{-}1\frac{1}{2} - {}^{-}2$
4. $^{-}1\frac{1}{2} - \frac{^{-}3}{4}$

E

1. Consider the points $^{-}10$ and 5 on a number line.
   a. Write two absolute value expressions to represent the distance between these two points.
   b. Evaluate both of your expressions. What is the distance between the points $^{-}10$ and 5 on a number line?
   c. Draw a number line to represent the distance you found in part (b).
2. Write two absolute value expressions for the distance between the two points on the number line below. Then evaluate your expressions.



F For parts (1)–(4), decide whether or not the expressions are equal.

1. $^{-}2 - {}^{+}3$ and $^{+}3 - {}^{-}2$
2. $^{+}12 - {}^{-}4$ and $^{-}4 - {}^{+}12$
3. $^{-}15 - {}^{-}20$ and $^{-}20 - {}^{-}15$
4. $^{+}45 - {}^{+}21$ and $^{+}21 - {}^{+}45$
5. Is there a Commutative Property of subtraction? Explain your answer.

ACE Homework starts on page 44.