Investigation 2

Adding and Subtracting Rational Numbers

In Investigation 1 you used number lines and chip boards to model rational numbers. Now, you will develop algorithms for adding and subtracting rational numbers.

An **algorithm** is a plan, or a series of steps, for doing a computation. In an effective algorithm, the steps lead to a correct answer, no matter what numbers you use. Your class may develop more than one algorithm for each operation. Set a goal to understand and skillfully use at least one algorithm for adding rational numbers and one algorithm for subtracting rational numbers.
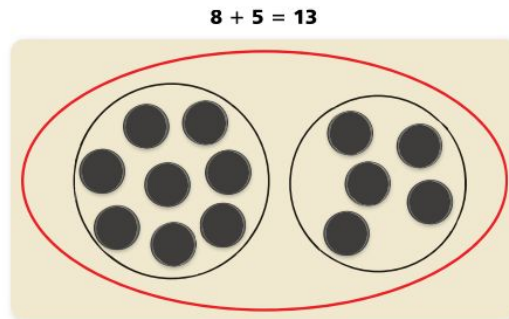
2.1 Extending Addition to Rational Numbers

There are two common ways that number problems lead to addition calculations like $8 + 5$. The first involves combining two similar sets of objects, as in this example:

> Linda has 8 video games, and her friend has 5.
> Together they have $8 + 5 = 13$ games.

You can represent this situation on a chip board.

$$8 + 5 = 13$$

Common Core State Standards

7.NS.A.1b Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses) . . .

7.NS.A.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Also 7.NS.A.1, 7.NS.A.1d, 7.NS.A.3, 7.EE.B.3

30   Accentuate the Negative

Number problems also lead to addition calculations when you add to a starting number. Here is an example:

At a desert weather station, the temperature at sunrise was 10°C. It rose 25°C by mid-day. The temperature at noon was 10°C + 25°C = 35°C.

You can represent this situation on a number line. The starting point is $^+10$. The change in distance and direction is $^+25$. The sum ($^+35$) is the result of moving a distance of 25 to the right.
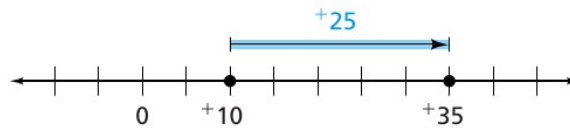
Suppose, instead of rising 25°C, the temperature fell 15°C. The next number line shows that $^+10°C + ^-15°C = ^-5°C$.
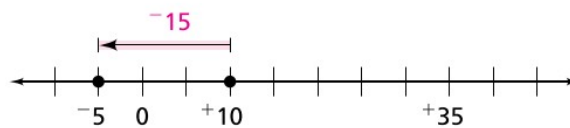
Suppose that the temperature change one day is $^-25°C$. What could the original temperature and the final temperature be for that day?

Use these ideas about addition as you develop an algorithm for addition of integers.

> (?) How can you predict whether the sum of two integers is 0, positive, or negative? Explain.

Problem 2.1

Use chip boards or number line models to solve these problems.

A **1.** Find the sums in each group.

Group 1
$^+2 + {}^+8$
$^-2 + {}^-8$
$^+8 + {}^+12$
$^-8 + {}^-12$

Group 2
$^+2 + {}^-8$
$^-2 + {}^+8$
$^+8 + {}^-12$
$^-8 + {}^+12$

 2. What do the examples in each group have in common?

 3. Write two new problems that belong to each group.

 4. Describe an algorithm for adding the integers in each group.

B You know that $^-5 + {}^-3 = {}^-8$. Use this information to help you solve the following related problems.

 1. $^-5\frac{1}{4} + {}^-3$

 2. $^-5\frac{1}{5} + {}^-3\frac{3}{5}$

 3. $^-5\frac{1}{3} + {}^-3\frac{2}{3}$

C You know that $^-8 + {}^+5 = {}^-3$. Use this information to help you solve the following related problems.

 1. $^-8.35 + {}^+5$

 2. $^-8.55 + {}^+5.3$

 3. $^-8.65 + {}^+5.25$

 4. Does your algorithm for adding integers from Question A work with fractions and decimals? Explain.

Problem 2.1 *continued*

D For parts (1)–(3), decide whether or not the expressions are equal.

1. $^-4 + {}^+6$ and $^+6 + {}^-4$
2. $^+2\frac{2}{3} + {}^-5\frac{7}{8}$ and $^-5\frac{7}{8} + {}^+2\frac{2}{3}$
3. $^-7\frac{2}{3} + {}^-1\frac{1}{6}$ and $^-1\frac{1}{6} + {}^-7\frac{2}{3}$
4. The property of rational numbers that you have observed in these pairs of problems is called the **Commutative Property** of addition. Explain why addition is commutative. Give examples using number lines or chip boards.

E 1. Find the sums in Group 3.
2. What do the examples in Group 3 have in common?
3. Write three new problems that belong to Group 3.

Group 3
$^-5 + {}^+5$
$^+9.4 + {}^-9.4$
$^+2\frac{1}{4} + {}^-2\frac{1}{4}$

F Write a story to match each number sentence. Find the solutions.

1. $^+50 + {}^-50 = \blacksquare$
2. $^-15 + \blacksquare = {}^+25$
3. $^-300 + {}^+250 = \blacksquare$

G 1. Use properties of addition to find each value.
   a. $^+17 + {}^-17 + {}^-43$
   b. $^+47 + {}^+62 + {}^-47$
2. Luciana claims that if you add numbers with the same sign, the sum is always greater than each of the addends. Is she correct? Explain.

A C E Homework starts on page 44.